RVARD COLLEGE
OBSERVATORY.

Cambridge, Dec. 10, 1877.

Sir,

Allow me to present to your Library the accompanying volume entitled Papers on Physics. Thirty-one copies ly of this work have been prepared, of hich this is Number twenty-five

Respectfully yours,

Edward C. Picke

To the Librarian of the Astor Library.

COMPILATION

OF THE

PAPERS ON PHYSICS

WRITTEN BY

harles

PROFESSOR EDWARD C. PICKERING.

1865-1877.

CAMBRIDGE:
1877.

For the convenience of those who may wish to consult my papers on Physics and kindred subjects, I give below a list of the more important. With these I have included the results of researches performed under my supervision in the Physical Laboratory of the Institute of Technology:—

Intersection of the Joints of an Oblique Bridge with the Plane of the Face in the English system. *Journ. Frank. Inst.*, l, 175 (Sept., 1865).

* Dispersion of a Ray of Light refracted at any number of Plane Surfaces. *Proc. Amer. Acad.*, vii, 478 (April, 1868).

* Essay on the Comparative Efficiency of different forms of the Spectroscope. *Amer. Journ. Sci.*, xlv, 301 (May, 1868).

* Description of a Machine for Drawing the Curves of Lissajous. *Journ. Frank. Inst.*, lvii, 55 (Jan., 1869).

Plan of the Physical Laboratory. (April, 1869.)

A New Form of Spectrum Telescope. *Engin. and Min. Journ.* (July, 1869).

Report on the Total Eclipse of August 7th, 1869. *Journ. Frank. Inst.*, lviii, 281 (Oct., 1869). Trans. into French in *Les Mondes*, xxi, 573.

* Observations of the Corona during the Total Eclipse. *Phil. Mag.*, xxxviii, 281 (Oct., 1869).

* Note on the Supposed Polarization of the Corona. *Journ. Frank. Inst.*, lviii, 372 (Dec., 1869).

* On the Diffraction produced by the Edges of the Moon. *Journ. Frank. Inst.*, lix, 265 (April, 1870).

* On the Focal Length of Microscope Objectives. By Chas. R. Cross. *Journ. Frank. Inst.*, lix, 401 (June, 1870).

Polarization of the Corona. *Nature*, iii, 52 (Dec., 1870).

Spectrum of the Aurora. *Nature*, iii, 104 (Dec., 1870).

List of Observations of the Polarization of the Corona. *Journ. Frank. Inst.*, lxi, 58 (Jan., 1871).

* The Graphical Method. *Journ. Frank. Inst.*, lxi, 272 (April, 1871).

* Photographing the Corona. *Journ. Frank. Inst.*, lxii, 54 (July, 1871).

* On Dispersion, and the Possibility of Attaining Perfect Achromatism. *Proc. Amer. Assoc.*, xix, 62 (Aug., 1871).

The Eclipse of 1870. *Old and New*, iii, 634 (May, 1871).

Report of Observations of the Total Eclipse of the Sun of Dec. 22d, 1870. *U. S. Coast Survey Report*, 1870, 115, 229.

* Report on the Physical Laboratory. 1871.

* A Geometrical Solution of some Electrical Problems. *Journ. Frank. Inst.*, lxvi, 13 (July, 1873).

On the Relative Efficiency of Kerosene Burners. By Chas. J. H. Woodbury. *Journ. Frank. Inst.*, xcvi, 115 (Aug., 1873).

* Applications of Fresnel's Formula for the Reflection of Light. *Proc. Amer. Acad.*, ix, 1 (Oct., 1873).

Measurements of the Polarization of Light reflected by the Sky and by one or more plates of glass. *Amer. Journ. Sci.*, cvii, 102 (Feb., 1874); *Phil. Mag.*, xlvii, 127 (Feb., 1874).

* Applications of the Graphical Method. *Proc. Amer. Acad.*, ix, 232 (May, 1874).

The Phonautograph. By Chas. A. Morey. *Amer. Journ. Sci.*, cviii, 130 (Aug., 1874).

* Graphical Integration. *Proc. Amer. Acad.*, x, 79 (Oct. 1874).

* I. Foci of Lenses placed obliquely. By Prof. E. C. Pickering and Dr. Chas. H. Williams. *Proc. Amer. Acad.*, x, 300.

* II. Light transmitted by one or more Plates of Glass. By W. W. Jacques. *Proc. Amer. Acad.*, x, 389.

* III. Intensity of Twilight. By Chas. H. Williams. *Proc. Amer. Acad.*, x, 421.

* IV. Light of the Sky. By W. O. Crosby. *Proc Amer. Acad.*, x, 425.

* V. Light absorbed by the Atmosphere of the Sun. By E. C. Pickering and D. P. Strange. *Proc. Amer. Acad.*, x, 428.

* VI. Tests of a Magneto-Electric Machine. By E. C. Pickering and D. P. Strange. *Proc. Amer. Acad.*, x, 432; *Electrical News*, i, 14, 54.

* VII. Answer to M. Jamin's objection to Ampère's Theory. By W. W. Jacques. *Proc. Amer. Acad.*, x, 445.

* VIII. An Experimental Proof of the Law of Inverse Squares for Sound. By Wm. W. Jacques. *Proc. Amer. Acad.*, xi, 265.

* IX. Diffraction of Sound. By Wm. W. Jacques. *Proc. Amer. Acad.*, xi, 269.

* X. Comparison of Prismatic and Diffraction Spectra. *Proc. Amer. Acad.*, xi, 273 (June, 1875).

* XI. On the Effect of Temperature on the Viscosity of Air. By S. W. Holman. *Proc. Amer. Acad.*, xii, 41.

* Mountain Surveying. *Proc. Amer. Acad.*, xi, 256 (Jan., 1876).

* Height and Velocity of Clouds. *Proc. Amer. Acad.*, xi, 263 (Jan., 1876).

* Progress of the Physical Department of the Mass. Inst. of Technology from 1867 to 1877.

Elements of Physical Manipulation. In 2 Vols. Vol. I, 1873, pp. 225. Vol. II, 1876, pp. 316.

Those marked in the above list with a star (*) have been bound in the present volume. Of the rest I have not a sufficient number of extra copies.

EDWARD C. PICKERING.

DESCRIPTION

OF A

MACHINE FOR DRAWING

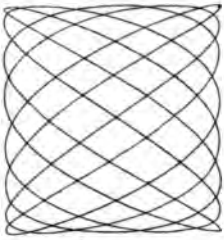
THE

CURVES OF LISSAJOUS.

BY PROF. EDWARD C. PICKERING,

Institute of Technology, Boston, Mass.

From the Journal of the Franklin Institute.



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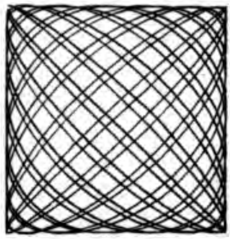

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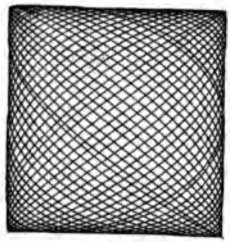

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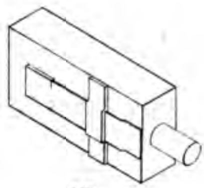

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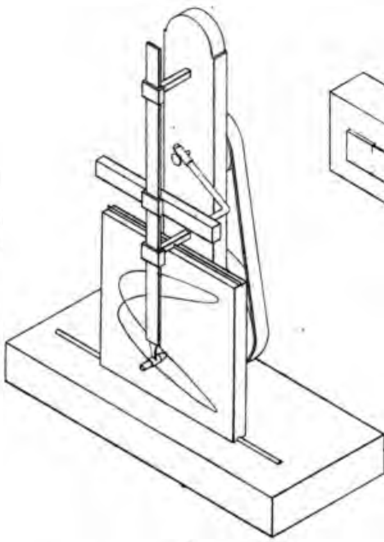
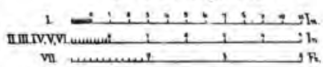


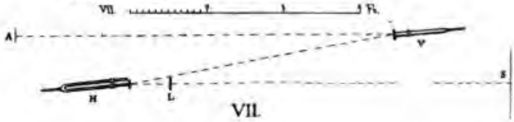
I.

A Machine for Drawing

LISSAJOUS' VIBRATIONS.

BY E. C. PICKERING.





VII.

DESCRIPTION

OF A

MACHINE FOR DRAWING

THE

CURVES OF LISSAJOUS.

BY PROF. EDWARD C. PICKERING,

Institute of Technology, Boston, Mass.

From the Journal of the Franklin Institute.

IN 1857, M. Lissajous studied the curves produced in the following experiment. Mirrors are attached to the prongs of two tuning forks, whose planes of vibration are at right angles to one another. A ray of light falling on the first, is reflected from it to the second, and is then projected on a screen by a lens. If, now, the fork which vibrates in a horizontal plane is sounded, the motion thus imparted to its mirror causes the luminous point on the screen to describe a horizontal line. Sounding the other fork in the same way, produces a vertical line. When both vibrate at the same time, various curves are produced, dependent on the relative pitch of the two forks.

When showing this experiment to an audience, it is desirable to have a set of curves, drawn on a large scale, for comparison. As, however, their geometrical construction is somewhat laborious, I have devised a machine, represented in the adjoining photo-lithograph, by which they may be drawn mechanically.

The paper on which the curves are drawn receives a horizontal motion to and fro, while, at the same time, the pen is moving vertically up and down. These motions are imparted thus:—Two

wheels (Fig. I.), connected by a belt, carry cranks, of which the lower one moves in a vertical slit in the rear of the square board to which the paper is attached. This board is kept in place below by a rod, over which it rolls, and above by a guide of sheet-brass. If, now, the lower wheel is turned, the paper moves horizontally through a distance equal to twice the crank-arm. A similar motion is imparted to the pen by a horizontal slit, in which the upper crank slides, and by which the pen is moved up and down. The rod is made flat to prevent its turning; or, it may be made circular, if we fix its slit in guides.

As a belt connects the two wheels, turning one moves both pen and paper. All the different phases of the curves corresponding to any given ratio of the number of vibrations of the two forks, may be drawn by merely sliding the belt over one of the wheels. As the ratio of the diameter of the latter corresponds to that of the number of vibrations of the forks, a series of wheels of different sizes are made, which, when combined two-and-two, give an almost endless variety of curves. Perhaps the best ratios, if we have four wheels, are $1 : 1\frac{1}{2} : 1\frac{3}{5} : 2$, by which we get curves corresponding to $\frac{16}{15}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, 2 and $\frac{8}{5}$, or to the semitone, major third, fourth, fifth, octave and minor sixth.

It may be proved that the curves coincide with those of Lissajous, since the angles described by the cranks are proportional to the diameter of the wheels; also, the distances of either pen or paper from the middle points of their paths are proportional to the sines of these angles. In other words, calling v the angle traversed by the upper crank, a, the ratio of the two wheels, and B, an angle dependent on the phase of the vibration, we have $y = \sin v$, and $x = (av + \text{B})$ equations defining Lissajous' curves.

To obtain curves corresponding to every ratio of the forks, expanding wheels may be used, in which, by turning a spiral guide, six sectors are made to approach or recede from the centre. With two such wheels, whose diameter can be altered from 3 to $4\frac{1}{4}$ inches, and from $4\frac{1}{4}$ to 6 inches, all the curves within the limits of an octave can be drawn. By a set screw the crank-arms may be shortened so that smaller curves may be drawn, also those inscribed in a rectangle instead of a square.

To draw the curves in ink, it was necessary to devise some new kind of pen. That represented in Fig. V*a*, answered as well as could be desired. It is easily made from a glass tube, by thicken-

ing and nearly closing one end with the blowpipe, placing a drop of water in it, and grinding it flat on a file. When filled, the convex liquid surface at the small end retains the ink, while, when pressed on the paper, the flow takes place with great smoothness. Fig. V*b*, shows a modification which is useful for drawing diagrams. The pen is made so as to contain a large supply of ink, which is kept from running out by closing one end by a cork, through which passes a tube, as in the figure. The pressure is then constant, and equals that of a column whose height is the distance from the point of the pen to the end of the inner tube. By varying this distance, we can use a high pressure for thick ink, and a low pressure for that which is more fluid.

In Figs. II., III. and IV., are given specimens of curves drawn by the apparatus above described, to show its practical working. In Fig. II., the ratio is 5 to 8, or the curve of the minor sixth; in Fig. III. it is 15 to 16, or a semitone; and in Fig. IV. about 3: 4, more nearly 44: 59, or a sharp fourth. In the latter case the curve does not close exactly, since the ratio is incommensurable. As these curves are here reproduced by photography, the slightest errors are visible, and they furnish a good test of the workmanship of the machine. Probably still greater accuracy would have been attained had the apparatus by which they were drawn been constructed by a professional instrument-maker.

When projecting the curves of Lissajous it is somewhat difficult to show the more complicated forms, owing to the large size of the luminous point compared with the curves themselves. I find the arrangement of Fig. VII. gives the best results. A is an aperture through which a ray of sunlight enters; V and H are the two forks, one vibrating in a vertical, the other in a horizontal plane; L is a lens of 6 feet focus, by which the curve is projected on the screen, S. The size of the curve depends only on the distance, L S, or on H S, if the light traverses the lens before meeting the mirror, H, and, therefore, the lens must be brought near the aperature, A. But, if we use a lens of short focal length, the luminous point is greatly magnified, since its diameter is proportional to $\frac{LS}{AV + VH + HS}$. I therefore use a lens of 6 feet focus, and remove V to a distance of $3\frac{1}{2}$ feet from H, so that $AV + VH + HL$ and LS shall be conjugate foci. This would be the case in Fig. VII., when L S is 18 feet, or in a room 20 feet wide. It is not represented in its true size, in the figure, for want of space.

Lenses suitable for these projections of any size not exceeding 8 inches in diameter, and 6 feet focus, may be obtained at a low price from the makers of cosmoramas. Although not achromatic, their aberration is exceedingly small, from their great focal length. They are useful for many purposes, particularly in large lecture-rooms; thus, in projecting the solar spectrum, I find them preferable to much more expensive achromatic lenses of short focal length. Again, by using one as an object-glass, a very good telescope for class purposes may be made for a few dollars, capable of showing all the more familiar astronomical objects, such as satellites of Jupiter, ring of Saturn, prominent nebulæ, &c., and which would, I think, be of great use to many institutions where a more expensive instrument could not be afforded.

Fig. VII. represents a substitute for the tuning forks in the experiment of Lissajous. It consists of a rectangular box, in which is an aperture just filled by a square plate, carrying the mirror. This is attached to an elastic strip of metal, which vibrates when air is forced into the box, like the tongue of a melodeon-reed. To vary the rapidity of vibration, the metal strip or tongue passes through a guide, by which means its length may be altered, and this may be done with the utmost precision by a screw not represented in the figure. This adjustment is not necessary in the reed replacing the second tuning fork, since a range of more than an octave is readily obtained by the method here described. The advantages claimed for this apparatus over the tuning forks are, that the angular amplitude is much greater, hence the curves on the screen are of very large size; that they may be maintained indefinitely by keeping up the supply by the bellows, and most important of all, by merely turning the screw, the progressive changes of one curve into another may be shown, so that, in a few moments, all possible combinations may be exhibited.

My experiments with this apparatus are not yet completed, as I find that the vibrating part must be made with a good deal of care, or the curve will not be perfect. In fact, in a preliminary experiment, in which I used too feeble a spring, I found that it answered admirably for showing the difference of quality of different sounds. By touching the mirror with the point of a knife, so as to produce a rattling noise, I obtained, with a single reed, instead of a right line, a complicated curve, which varied with every change in the sound.

It was of slow growth, planting its roots far back in the ages of barbarism, — a final result, to which the experience of the ages had steadily tended. The family, which in this view of the case is essentially modern, is the offspring of this vast and varied experience of the ages of barbarism.

Since the family was reached, it has also had its stages of progress, and a number of them. The rise of family names, as distinguished from the single personal name common in barbarous nations, is comparatively modern in the Aryan family. The Roman GENS is one of the earliest illustrations. This people produced the triple formula to indicate the *name of the individual*, of the *Gens* or *great family*, and of the *particular family* within the *Gens*. Out of this arose, in due time, the doctrine of agnation, to distinguish the relationship of the males, who bore the family name, from that of the females of the same family. Agnatic relationship was made superior to cognatic, since the females were transferred, by marriage, to the families of their husbands. This overthrew the last vestige of tribalism, and gave to the family its complete individuality.

15. The Overthrow of the Classificatory System of Relationship and the Substitution of the Descriptive. — Without attempting to discuss the fragments of evidence tending to show that the Aryan, Semitic, and Uralian families once possessed the classificatory system, it will be sufficient to remark, that, if such were the fact, the rights of property and the succession to estates would have insured its overthrow. These are the only conceivable agencies sufficiently potent to accomplish so great a change. Without such a change the family, as now constituted, would have remained impossible.

In conclusion I may remark, that the probable truth of this solution cannot be fully appreciated from the limited presentation of the facts contained in this article. At most it will but serve to invite attention to the great sequence of customs and institutions which seem to mark the successive stages of man's progress through the periods of barbarism, and to indicate the intimate relations which this remarkable system of consanguinity appears to sustain to the condition, experience, and advancement of mankind during the primitive ages. The manuscript containing the body of the evidence is now in course of publication by the Smithsonian Institution.

Five hundred and ninety-second Meeting.

March 10, 1868. — ADJOURNED STATUTE MEETING.

The PRESIDENT in the chair.

The President called the attention of the Academy to the recent decease of Sir David Brewster of the Foreign Honorary Members, and of Hon. Daniel Lord, of New York, of the Associate Fellows.

Five hundred and ninety-third Meeting.

April 14, 1868. — MONTHLY MEETING.

The PRESIDENT in the chair.

The President called the attention of the Academy to the recent decease of Dr. Samuel L. Dana, of the Resident Fellows, and of Professor William Smyth, of the Associate Fellows.

The following paper was presented: —

*Dispersion of a Ray of Light refracted at any number of Plane Surfaces.** By EDWARD C. PICKERING.

LET $a_1\ a_2\ a_3$, &c., be the angles included between the surfaces, $n_1\ n_2\ n_3$ their indices of refraction, $i_1\ i_2\ i_3$ the angles of incidence, $r_1\ r_2\ r_3$ the angle of refraction; $\sin i_1 = n_1 \sin r_1$ and in general

$$\sin i_m = r_m \sin r_m \tag{1}$$

also

$$i_m = a_{m-1} + r_{m-1} \tag{2}$$

As the dispersion of any portion of the spectrum is always proportional to the angular divergence of two rays of nearly equal refrangibility, if we vary $n_1\ n_2$, &c., dr_m will measure the dispersion. Differentiating (1)

$$\cos i_m\, di_m = r_m \cos r_m\, dr_m + \sin r_m\, dn_m$$

* Since presenting this communication to the Academy, I have learned that a portion of this subject was studied by Sir David Brewster in 1812. I have, therefore, modified my paper, omitting what was not new, except when necessary to preserve the context.

if $n_1\ n_2\ n_3$ are all functions of N, making the latter the independent variable and dividing by dN, we have

$$\frac{dr_m}{dN} = \frac{\cos i_m}{n_m \cos r_m}\frac{di_m}{dN} - \frac{\sin r_m}{n_m \cos r_m}\frac{dn_m}{dN} \tag{3}$$

But differentiating (2)

$$di_m = dr_{m-1}$$

and calling

$$\frac{\cos i_m}{n_m \cos r_m} = a_m, \quad \frac{\sin r_m}{n_m \cos r_m}\frac{dn_m}{dN} = b_m$$

also the dispersion of a ray after passing m surfaces, or

$$\frac{dr_m}{dN} = l_m,$$

and (3) becomes

$$l_m = a_m\, l_{m-1} - b_m \tag{4}$$

This formula by successive substitutions may be applied to any case. For a single surface

$$m = 1,\ l_0 = 0,$$

and hence

$$l_1 = a_1\, 0 - b_1 = -\frac{1}{n_1} \operatorname{tang} r_1$$

which equals unity when tang $r_1 = n_1$ or at the angle of total polarization. That is, the unit of dispersion is that produced by a single surface when the ray is in the position of total polarization. For two surfaces (4) becomes

$$l_2 = a_2\, l_1 - b_2 = -(a_2\, b_1 + b_2);$$

in a prism $n_2 = \frac{1}{n_1}$ making suitable substitutions, and reducing we obtain

$$l_2 = \frac{\sin a}{\cos r_1 \cos r_2}$$

a being the angle of the prism. For minimum deviation

$$r_1 = \frac{a}{2},\ r_2 = i_1 \text{ and } l_2 = \frac{2}{n} \operatorname{tang} i;$$

can only be used by identifying the more prominent lines, with those whose wave-length is known, and interpolating the remainder approximately. If, as is often the case, these standard lines cannot be recognized, the photograph becomes useless. To show the amount of distortion, suppose a spectrum to contain three similar double lines *A*, *B*, and *C*, whose indices of refraction are 1.5, 1.6, and 1.7, and that we use a 60° prism, the line *B* being in the position of minimum deviation. The deviations of the three lines will then be 48° 34′, 53° 8′, and 58° 11′, and their dispersion 1.528, 1.667, 1.878; that is, *A* and *C* will be at distances of 4° 34′ and 5° 3′ from *B* instead of equidistant, and the interval between the components of each line will be as 1.528 : 1.667 : 1.878; the distortion in this case amounting to about 20 per cent.

If now the portion of the screen which receives the line *C* be brought nearer the prism, the parts of this line will approach one another, and since their distance apart is proportional to their distance from the prism, the three lines will appear alike, if the screen is so inclined that the points where they are projected are at distances,

as $$\frac{1}{1.528} : \frac{1}{1.667} : \frac{1}{1.878}$$

or as $$.654 : .600 : .532.$$

A simple calculation shows that the screen must be slightly curved to fulfil this condition, but if plane, the distortion will be only about one and a half, instead of twenty per cent, the angle of inclination with the ray *B* being about 37°. If an achromatic lens was used for the projection, all parts of the spectrum would not be in focus, but with a single lens the focal distance of the violet is always less than that of the red rays. If then we use such a lens, inclining the screen at the same time corrects the distortion, and brings all parts into focus at once. By placing the prism at a suitable distance from the lens, both sources of error may be almost entirely eliminated. The oblique incidence of the light on the sensitive surface would be an objection to this method, but would be partly counterbalanced by the fact that the length of the spectrum would be thereby increased more than one half. Or, if preferred, the prism may be turned, so that, applying the correction for distortion as above, the screen shall be more nearly perpendicular to the light.

In conclusion, this spectrum would possess the following advantages

over the distorted forms now in use. Horizontal distances being proportional to the change in the index of refraction, the latter could be determined at once for any line, by a scale of equal parts. Its extent would be much greater than that of the visible spectrum, and we could determine the index of refraction of rays of too short wave-length to be measured readily by the common methods. It would be a normal spectrum for any given material, being independent of the form and position of the prism. And (especially if the interference bands were produced in it) it would afford, from its extent, great advantages for the study of the laws of the dispersion of light by different substances.

Five hundred and ninety-fourth Meeting.

May 12, 1868. — Monthly Meeting.

The President in the chair.

The Corresponding Secretary read letters relative to exchanges; also a letter from Professor De La Rive in acknowledgment of his election into the Academy as a Foreign Honorary Member.

Mr. C. M. Warren presented by title a memoir on "Volatile Hydrocarbons in Pennsylvania Petroleum."

DONATIONS TO THE LIBRARY,

FROM JUNE 2, 1865, TO JUNE 30, 1866.

State of Massachusetts.

Report to His Excellency the Governor and the Honorable Council, of the Commissioners appointed under the Resolve of May 3, 1865, "concerning the Obstructions to the Passage of Fish in the Connecticut and Merrimack Rivers." 8vo pamph. Boston. 1866.

Massachusetts Historical Society.

Proceedings. 1864 – 1865. 8vo. Boston. 1866.

Massachusetts Institute of Technology.

First Annual Catalogue of the Officers and Students, and the Programme of the Course of Instruction, of the School of the Massachusetts Institute of Technology. 1865 – 66. 8vo pamph. Boston. 1865.

Boston Society of Natural History.

Proceedings. Vol. IX., X. 8vo. Boston. 1865 - 66.

Condition and Doings of the Boston Society of Natural History, as exhibited by the Annual Reports of the Custodian, Treasurer, Librarian, and Curators. May, 1865. 8vo pamph. Boston. 1865.

Trustees of the Public Library of the City of Boston.

Thirteenth Annual Report of the Trustees, 1865. 8vo pamph. Boston. 1865.

Boston Athenæum.

List of Books added to the Library of the Boston Athenæum, from December 1, 1864, to December 1, 1865. 8vo pamph. Boston. 1865.

Observatory of Harvard College.

Report of the Committee of the Overseers of Harvard College, appointed to visit the Observatory, in the Year 1864. Together with the Report of the Director. Submitted March 8, 1865. 8vo pamph. Boston. 1865.

Director of the Museum of Comparative Zoölogy.

Illustrated Catalogue of the Museum of Comparative Zoölogy at Harvard College. Published by Order of the Legislature of Massachusetts. No. I. Ophiuridæ and Astrophytidæ. By Theodore Lyman. No. II. North American Acalephæ. By Alexander Agassiz. 2 vols. 4to. Cambridge. 1865.

Annual Report of the Trustees of the Museum of Comparative Zoölogy. Together with the Report of the Director, 1865. 8vo pamph. Boston. 1865.

Essex Institute.

Proceedings. Vol. IV. 8vo. Salem. 1866.

An Historical Notice of the Essex Institute; with the Act of Incorporation, Constitution and By-Laws, and Lists of the Officers and Members. 8vo pamph. Salem. 1865.

Naturalists' Directory. Part I. North America and the West Indies. 8vo pamph. Salem. 1865.

American Antiquarian Society.

Proceedings of the American Antiquarian Society at a Special Meeting, January 17, 1865, in reference to the death of their former President, Hon. Edward Everett. 8vo pamph. Boston. 1865.

Proceedings at the Semiannual Meeting held in Boston, April 26, 1865. 8vo. Boston. 1865.

ESSAY

ON THE

COMPARATIVE EFFICIENCY

OF

SPECTROSCOPE PRISMS OF DIFFERENT ANGLES.

By EDWARD C. PICKERING,
Assistant Prof. of Physics, Mass. Inst. of Technology.

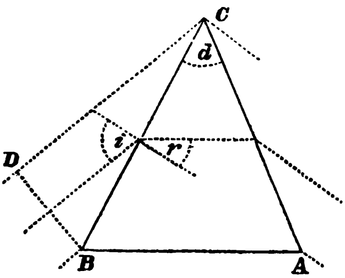
[FROM THE AMERICAN JOURNAL OF SCIENCE AND ARTS, VOL. XLV, MAY, 1868.]

ON THE

COMPARATIVE EFFICIENCY OF DIFFERENT FORMS OF THE SPECTROSCOPE.

BY EDWARD C. PICKERING.

IT is the object of the present paper to furnish a means of comparing with accuracy spectroscope prisms of different forms, and to determine what must be their refracting angle, to produce the greatest dispersion, with the least loss of light. We have then to consider the dispersion, the loss by reflection, and that by absorption.



1. *Dispersion.*—The dispersion of any part of the spectrum, is proportional to the angular interval between two rays of nearly equal refrangibility, as the two parts of a double line.

Let α be the refracting angle of the prism ABC, n the index of refraction of the less refrangible ray. For minimum deviation, $r=\frac{\alpha}{2}$, $\sin i = n \sin\frac{\alpha}{2}$ . . . . . (1). If dn be the difference of the indices of refraction of the two rays, di will be their angular divergence. Differentiating (1), $di=\frac{\sin\frac{\alpha}{2}}{\cos i}dn=\frac{1}{n}\tan i\, dn$ .. (2), in which $\frac{1}{n}$ tang i serves as a measure of the dispersion under different angles of incidence. It differs essentially (when the angle of incidence is large) from the deviation which is commonly, but incorrectly, assumed as the measure.

Comparative dispersion and deviation of a ray entering a medium in which n=1·5.

Angle of incidence i,	0°	15°	30°	45°	56° 19′	60°	75°	80°	85°	90°
Dispersion $\frac{1}{n}$ tang i,	·0	·179	·385	·667	1·000	1·1565	2·488	3·781	7·620	∞
Deviation $i-r$,	0°	5° 4′	10° 32	16° 53′	22° 38′	24° 44′	34° 55′	38° 58′	43° 23′	48° 11′
" reduced to same unit as dispersion,	·0	·231	·465	·746	1·000	1·093	1·543	1·720	1·917	2·121

The dispersion then increases much more rapidly than the deviation; hence in spectroscopes whose deviation is the same, that one will disperse most, in which i and therefore α is the greatest.

The above discussion applies strictly only to the emergent ray, but in the position of minimum deviation, the dispersion of a prism is just double this, as may be seen from the general formula for dispersion (Radicke's Optics, vol. i, p. 179). I propose hereafter to discuss the question whether greater dispersion with the same loss of light could not be obtained by some other position of the prism.

2. *Reflection.*—In estimating the loss by reflection it is usual to assume that the same proportion of the incident light is lost at each successive refraction. But in reality the light so refracted is partially polarized, and in this state another law determines the amount reflected. Fresnel showed that of a ray polarized in the plane of incidence, the proportion reflected $B' = \frac{\tan^2(i-r)}{\tan^2(i+r)}$, while a ray polarized in a plane perpendicular to the first, would lose by reflection $A' = \frac{\sin^2(i-r)}{\sin^2(i+r)}$.

Regarding common light as composed of two beams of equal intensity polarized at right angles, the amount reflected would be $\frac{1}{2}A' + \frac{1}{2}B'$, and that transmitted $\frac{1}{2}[(1-A')+(1-B')]$. On meeting a second surface inclined at the same angle of incidence, the amount transmitted would be $\frac{1}{2}[(1-A')^2+(1-B')^2]$ and after passing m surfaces $\frac{1}{2}[(1-A'^m)+(1-B')^m]$.

This formula can be applied directly to the m surfaces of the prisms of a spectroscope, since in the position of minimum deviation i and r are the same for all, and therefore the amount transmitted is the same, whether the passage is from glass to air, or from air to glass.

The formulas of Fresnel are used in preference to those of Cauchy, although the latter have been proved, by Jamin and others, to be more correct. But the coefficient of ellipticity which they involve is neither so commonly, nor so easily found as the index of refraction. Furthermore, for glass the difference would probably be so small that it could be neglected.

3. *Absorption.*—The average length of glass traversed by the light is one half the base AB, multiplied by N the number of prisms, and the amount escaping absorption is proportional to the log. of this distance, or to $\log N \times BC \sin\frac{1}{2}\alpha$, or in prisms admitting the same amount of light (that is in which BD is the same) to $\log BD \times N \frac{\sin\frac{1}{2}\alpha}{\cos i}$, since $BC = \frac{BD}{\cos i}$, but the dispersion

is proportional to $N\frac{\sin\frac{1}{2}\alpha}{\cos i}$, hence in spectroscopes dispersing equally and composed of prisms of the same material, *the loss by absorption will be the same in all*, so that as far as the absorption is concerned, it makes no difference whether a spectroscope is composed of a large number of very acute angled prisms, or of a less number in which the angle is more obtuse.

Thus we avoid a difficulty which seemed at first sight insurmountable, since the actual amount of light absorbed varies not only with the material, but with the refrangibility of the rays, and according to laws not yet discovered.

The following tables give the deviation, dispersion, and amount of light escaping reflection, of spectroscopes composed of from one to ten prisms of indices of refraction 1·5, 1·6 and 1·7.

Table I applies to prisms of 45°. Table II to those of 60°, these being the forms in general use, and Table III. where the angle is such that the reflected light would be totally polarized, α being 67° 22′, 64°, and 60° 56′ in the three cases respectively. This form of prism appears to present great advantages for large spectroscopes, since at most but one half of the light can be reflected, while one prism disperses as much as two of 45°.

TABLE I.—45° *Prisms.*

	n	1 surface.	1 prism.	2 p.	3 p.	4 p.	5 p.	10 p.
Deviation	1·5	12° 32′	25° 4′	50° 8′	75° 12′	100° 16′	125° 20′	250° 40′
$i-\frac{\alpha}{2}$	1·6	15° 15′	30° 30′	61° 0′	91° 30′	122° 0′	152° 30′	305° 0′
	1·7	18° 5′	36° 10′	72° 20′	108° 30′	144° 40′	180° 50′	361° 40′
Dispersion	1·5	·467	·935	1·870	2·804	3.739	4·674	9·348
$\frac{\sin\frac{\alpha}{2}}{\cos i}$	1·6	·484	·968	1·936	2·904	3·872	4.840	9·680
	1·7	·504	1·008	2·016	3·023	4·031	5·039	10·078
Transmitted	1·5	·957	·916	·841	·774	·724	.661	·461
$\frac{1}{2}[(1-A')^m+$	1·6	·943	·892	·799	·719	·651	·592	·391
$(1-B')^m]$	1·7	·926	·859	·745	·653	·578	·516	·324

TABLE II.—60° *Prisms.*

		1 surface.	1 prism.	2 p.	3 p.	4 p.	5 p.	10 p.
	1·5	18° 35′	37° 10′	74° 20′	111° 30′	148° 40′	185° 50′	371° 40′
Deviation	1·6	23° 8′	46° 16′	92° 32′	138° 48′	185° 4′	231° 20′	462° 40′
	1·7	28° 13′	56° 26′	112° 52′	169° 24′	225° 44′	282° 10′	564° 20′
	1·5	·756	1·512	3·023	4.535	6·046	7·558	15·116
Dispersion	1·6	·833	1·667	3·334	5·000	6·667	8·334	16·668
	1·7	·949	1·899	3·797	5·696	7·594	9·493	18·986
	1·5	·945	·895	·811	·742	·686	·641	·509
Transmitted	1·6	·920	·853	·748	·672	·618	·578	·491
	1·7	·888	·801	·681	·608	·565	·538	·505

TABLE III.—*Angles of Prisms* 67° 22′, 64° *and* 60° 56′.

Deviation	1·5	22° 38′	45° 16′	90° 32′	135° 48′	181° 4′	226° 20′	452° 40′
	1·6	26°	52°	104°	156°	208°	260°	520°
	1·7	29° 4′	58° 8′	116° 16′	174° 24′	232° 32′	290° 40′	580° 20′
Dispersion	1·5							
	1·6	1.	2.	4.	6.	8.	10.	20.
	1·7							
Transmitted	1·5	·923	·863	·763	·691	·639	·600	·520
	1·6	·899	·818	·702	·629	·582	·552	·505
	1·7	·874	·780	·657	·588	·549	·523	·501

To apply these tables to an example, let us compare three spectroscopes of ten prisms each of angles 45°, 60° and 64°, the index of refraction being 1·6.

10 prisms of	Deviation.	Dispersion.	Transmitted.	Trans. × cos i.
45°	305° 0′	9·680	·3911	·308
60°	462° 40′	16·668	·4912	·294
64°	520°	20·000	·505	·268

Again, comparing spectroscopes producing equal deviation,

	Deviation.	Dispersion.	Transmitted.	Trans. × cos i.
12 prisms of 45°	366°	11·616	·339	·268
8 " " 60°	370° 8′	13·334	·532	·319
7 " " 64°	364°	14.	·521	·276

From the first example we see that by using 64° prisms instead of 45° we obtain more than double the dispersion, with even less loss of light, while in the second case seven 64° prisms prove much more efficient than twelve of 45°.

All these calculations seem to point to the superiority of 60° prisms, over those of 45°. A much greater angle is objectionable from the increased distortion produced by the slightest imperfection in the refracting faces.

In prisms admitting the same amount of light the more acute the angle, the less is the quantity of glass and the less the area of each face. The ground might be taken that a 45° prism could be made larger than one of 60° at the same expense, and thus the difference in light remedied. In this case, however, it would be necessary to enlarge the telescope, number of prisms, and in fact the whole instrument. Even supposing this change made, the prisms of larger angle preserve their superiority, though not in so marked a degree. The calculation is readily made by multiplying the transmitted light by cos i, as is done in the above examples.

The index of refraction varying with the refrangibility of the rays, the dispersion, loss of light, &c., would vary in different parts of the spectrum. The change would, however, be small, and could be determined, if necessary, by merely altering n.

Institute of Technology, Boston, Feb. 29th, 1868.

From the PHILOSOPHICAL MAGAZINE for October 1869.

OBSERVATIONS OF

THE CORONA DURING THE TOTAL ECLIPSE,

AUGUST 7, 1869

BY

PROFESSOR EDWARD C. PICKERING.

AMONG other expeditions to observe the recent eclipse was one under the direction of Professor Henry Morton, sent by the Nautical-Almanac Office to photograph the sun. I was attached to this party to make general and physical observations, and from our station at Mount Pleasant, Iowa, arrived at the following results.

It is commonly supposed that the light of the corona is polarized in planes passing through the sun's centre, and that it shines by reflected light. Wishing to verify this observation, I prepared an Arago's polariscope (in which the objects are viewed through a plate of quartz), and a double-image prism of Iceland spar. The two images appear of complementary colours when the light is polarized, the tint changing with the plane of polarization. I therefore expected to see two coloured coronas, the tint of each portion being complementary to that of the part at right angles to it, and the colour revolving with the polariscope. In reality the two images were pure white without any traces of colour; but the sky adjoining one was blue, adjoining the other yellow. As the instrument is of considerable delicacy, we must conclude that little or no polarized light is emitted by the corona. The sky adjoining it, however, is polarized in a plane in-

dependent of the position of the sun, since its colour (as seen in the polariscope) is the same whether above, below, or on one side of it. The most probable explanation of this curious phenomenon is, that the earth beyond the limits of the shadow, being strongly illuminated, acts as a new source of light, and thus gives rise to a polarization in a plane perpendicular to the horizon.

In hopes of determining the cause of discrepancy between this observation and those previously made, I have endeavoured to learn what form of polariscope has heretofore been used; but, unfortunately, in most cases no description has been published. One observer used a Savart's polariscope, and, holding it with its principal plane vertical, found strong traces of polarization in this plane. This observation, however, agrees with mine if we suppose that the polarization of the sky was taken for that of the corona, a natural mistake with this form of instrument. Another observer, who used a single plate of tourmaline, saw no evidence of polarization, that of the sky being too feeble to be perceived in this way. I verified my results with a simple prism of Iceland-spar, with which two images of the corona were seen precisely alike and showing no signs of polarization. We cannot infer from this that the corona is self-luminous, since polarization is produced only by specular and not by diffuse reflection.

The spectrum of the corona was observed in the following manner. A common chemical spectroscope was used; but instead of attaching it to a telescope, it was merely pointed in the proper direction a short time before totality. As its field of view was 7 or 8 degrees in diameter, the sun remained in it for a considerable time, and the spectrum obtained was that due to the corona, protuberances, and sky near the sun. On looking through the instrument during totality, a continuous spectrum was seen free from dark lines, but containing two or three bright ones—one near E, and a second near C. At the time, I supposed that these were due to the protuberances; but Professor Young, with a large spectroscope of five prisms, found a line near E which remained visible even when the image of the protuberance was moved off the slit, and therefore inferred that it was due to the corona. He also found the continuous spectrum free from dark lines—and that one, perhaps three of the bright lines coincide with those of the aurora borealis. These results would lead to the belief that the corona is self-luminous, the bright lines rendering its gaseous nature probable. If it is a part of the sun, even the remoter portions are one hundred times as near as the earth, and would receive ten thousand times as much heat, which would be sufficient to raise any known substance to incandescence.

Other observations, however, point to quite a different conclusion. A thermometer with blackened bulb was exposed to the sun's rays and the temperature recorded every five minutes. I found that it began to rise some time before contact, descending again as soon as the moon's limb became visible. It did not reach its former temperature until about a quarter of an hour after the eclipse began, or until a seventh of the sun's disk was obscured. The approach of the moon, therefore, appeared to cause an *increase* in the sun's heat. The amount of the change was only about $1^{\circ}\cdot3$ C., the total difference between this thermometer and one in the shade being about 18° C., or in the ratio of 1 to 14. This fraction is but one-half of that given above, owing perhaps to the diminution of heat on the borders of the sun. During totality the difference between the two thermometers was almost nothing. In examining the photographs taken by the party, it was noticed that, while the light diminished near the edge of the sun, the moon's limb was very distinct, and that there was a marked increase in the light of the parts nearest it. It was suggested that this might be a subjective effect; but an examination of the photographs is sufficient to convince any one that the appearance is a real one. The glass positives especially show that this effect extends over a large part of the sun's disk. The exposure was rendered instantaneous by passing a diaphragm with a slit in it in front of the camera, the rapidity of motion being regulated by a series of springs. Any irregularity in the motion would cause variations in shade in the photographs; but these would form bands parallel to the slit, while the shade mentioned above was not parallel to it and was curved so as to follow the moon's edge. Since, then, there is an increase both of the actinic power and of the heat, it would seem that these effects are real, since the methods of observing them are so totally different that no error in one could be introduced into the other. The only explanation of the phenomenon that seems possible is to assume the presence of a lunar atmosphere. The corona would then be caused by refraction, light reaching the observer from parts of the sun already eclipsed. Although for various reasons this hypothesis is unsatisfactory, yet it is strengthened by other observations. The protuberances have often seemed to indent the moon's edge, an appearance usually ascribed to irradiation. Several of the photographs, however, show this same effect; and in some of them the exposure was so short and the edges of the protuberances are so well defined that it cannot be caused by the intensity of their light, but must have its origin outside of the eye of the observer. It is noticeable on all sides of the moon, sometimes in half a dozen protuberances in a single photograph. An atmosphere of rapidly increasing

density might produce this effect by reflection, and of course would not influence the corona if it was caused by refraction. On this supposition reliance could not be placed on measurements of the moon's diameter by occultations, or by contacts during eclipses, and would account for the uncertainty of this constant.

The principal reason for supposing the corona a portion of the sun is, that during totality it does not appear to move with the moon, but remains concentric with the sun, or, more properly, is brightest where the sun's edge is nearest. Many of the photographs show this very well, the difference on the two opposite sides of the moon being very marked. Now this effect would be explained equally well by supposing the corona caused by refraction. For the centres of the sun and moon never differ during totality by more than half a digit, while the breadth of the corona is sometimes several times as much; so that merely covering a small portion of it would not produce a greater diminution of light than would be caused by a slight change in the direction of the sun's rays shining through a lunar atmosphere. On the other hand, it is difficult to conceive of an atmosphere dense enough to produce these effects, and yet so transparent that the edges of the full moon are perfectly distinct, and that the light of the sun during an eclipse should be increased rather than diminished. Again, we should expect that such variations would be produced by changes of temperature that they could scarcely fail to be detected.

We then conclude that the polariscope gives only negative results, and cannot be regarded as proving that the light is reflected. The evidence of the spectroscope needs confirmation, since the dark lines may have been invisible owing to the feeble light of the corona. But if the observations with it are correct, the self-luminous character of the corona is established. The thermometric and actinic experiments point towards a lunar atmosphere as the cause of the corona.

In the above I have endeavoured to give the evidence in favour of each view, unbiased by any theory, leaving to those best able to judge to determine whether either explains all the facts observed. The absence of a lunar atmosphere is so generally admitted, that its existence is suggested only with reluctance, and merely as the most natural explanation of the observations.

Boston, U.S., Sept. 1, 1869.

Note on the Supposed Polarization of the Corona.

By Prof. E. C. Pickering.

An observation on this subject is given in my report on page 285 of the current volume of this *Journal*, but as the form of the instrument used has been in one or two cases misunderstood, I enclose a sketch of it. A B (Fig. 1) is a sheet-iron tube, closed at A with a plate of quartz and at B with a prism of Rochon. The latter has the property of giving two images of any object seen through it, separated by an angle of nearly 3°. Looking through the tube we therefore see two images of the quartz touching, but not overlapping. When the light is polarized these images assume complementary tints, which vary with the plane of polarization and the thickness of the quartz. On turning this instrument towards the sun during totality, the images presented the appearance shown in Fig. 2.

Fig. 1.

The hexagons represent in form and size the plate of quartz; the black circles the moon, here drawn a sixth of an inch in diameter, as the scale is about 3° to the inch. The corona appeared white, but the sky surrounding it was colored in one image blue, in the other, yellow,—represented in the figure by vertical and horizontal lines. The conclusion to be drawn from this is, that the light of the corona is unpolarized, or, more strictly, that the amount of polarized light, if any, is too slight to be perceptible with this instrument. Its delicacy, although not equal to Savart's polariscope, is very great, giving colored images with paper, wood and other bodies which reflect a small amount of light specularly. The day before the eclipse it showed, in a very marked manner, the polarization of the wet pavements and roofs. To measure its

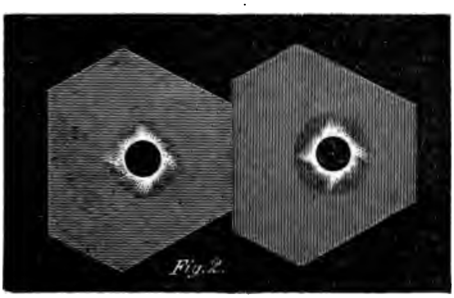


Fig. 2.

sensitiveness, I viewed the light reflected by a piece of plate glass, at different angles of incidence, and found that the color ceased to be visible when this angle was about 10°, which, allowing for the reflection from the second face would give about one part of polarized to twenty-four of natural light.

Observers heretofore have generally attached their polariscope to a telescope, and thus introduced a source of error, avoided in my instrument. For, light passing through the object-glass and field lens, would be polarized by refraction before reaching the polariscope by the obliquity of the incidence, caused both by the curvature of the surfaces and the fact that the edge of the field of view receives its light not parallel to the axis. The plane of polarization would be perpendicular to a plane falling through the axis of the instrument. Now, if any part of the corona was brought into the centre of the field of view, the adjoining portions would appear polarized in planes parallel to the edge of the field, or passing through the sun's centre. In sweeping around the sun's edge the plane of polarization would continually change, as the corona passed through different parts of the field, and the comparative darkness of the moon's disk and the exterior sky prevent the polarization of the other portions of the field from being visible. The degree of polarization by refraction would be very slight and perhaps imperceptible, but the agreement of observation with this hypothesis is certainly a curious coincidence.

The strongest argument against the polarization of the corona is furnished by the spectroscope, the presence of bright lines and absence of dark ones, as observed by Prof. Young, denoting incandescence, a view strengthened by the consideration that each square centimetre of the surface of the corona would receive several thousand units of heat per minute. I am well aware that my results are at variance with those obtained by previous observers, including some of the most eminent astronomers of the day, but as far as I can learn this form of polariscope has not been used for the purpose, and therefore hope that my experiment may be repeated during the next eclipse.

NOTE. Since writing the above I learn from Prof. F. H. Smith that an excellent Arago's polariscope was used at Eden Ridge, Tenn., in observing the eclipse. The result agreed with mine, namely, that no traces of polarization could be detected in the corona with this instrument. E. C. P.

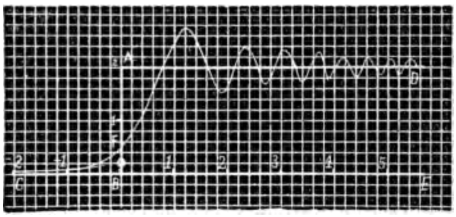
ON THE DIFFRACTION PRODUCED BY THE EDGES OF THE MOON.

By Prof. Edward C. Pickering.

In the report of Dr. Curtis on the eclipse of last summer, he describes some experiments tending to show that the brightness seen in the photographs along the moon's limb is caused by diffraction. He adds also a note by President Barnard explaining the same phenomenon geometrically. On the other hand, the experiments of Prof. Morton show pretty conclusively that it is a chemical effect produced in developing the photograph. Moreover, Prof. Smith although looking carefully for this appearance with a good telescope, was unable to detect it by the eye. The undulatory theory is so well founded that any conclusions derived from it would have great weight; it therefore appeared necessary to examine, critically, the mathematical discussion mentioned above, while the eminence of the physicist who has propounded it, requires that I should give somewhat fully the steps by which I have arrived at quite an opposite result. The argument used by President Barnard is this:—along the edge of the shadow, fringes are formed, alternately bright and dark; but there is a greater increase in brilliancy in the first bright band than diminution in the first dark one, and the same is true of the second, third and of all following it. Hence, he concludes, there is an increase in the total amount of light. Suppose this to be true, and that a luminous point is placed in the centre of a spherical room containing a mirror. We commonly assume that the light cut off from one side is reflected to the other, so that the total illumination is the same as if the mirror was not present. But according to the above theory since diffraction would take place on each edge, both of the mirror's shadow and of the reflected beam, we should obtain an increase of light at all these points. By merely inserting more mirrors we could thus indefinitely increase the illumination, a result quite at variance with the theory of the conservation of forces.

The error, as it seems to me, is that no account is taken of the diminution of light close to the edge of the geometrical shadow. Fresnel computed the intensity of the light at 126 different points, and has represented the results by a sinuous curve E F D, in which abscissas correspond to distances from the geometrical shadow A B,

and ordinates to the intensity of the light.* If there was no dif-fraction the intensity would everywhere equal 2 or A B, and its total quantity would be represented by the rectangle A B E D. The problem then is to compare this area with that included between the curved line and E B C. I have been unable to find any such comparison, and since the formulas are too complex to render an integration possible, I resorted to a measurement of the curve, constructing it on a scale 20 times that of the above figure. The part



B F C corresponds to the light which is inflected or bent within the shadow, and has B C for an asymptote. The area of the part within the limits discussed by Fresnel, that is from 0 to — 5·5 can be obtained with a good deal of accuracy, and gives the result ·300; but beyond this we have a space whose height on the above scale nowhere exceeds one thousandth of an inch, but whose area might be very considerable, since its length is indefinitely great. To determine this area, at least approximately, I devised the following method which I hope may be found useful in many similar cases. Let $y' = f(x')$ be the equation of the curve of which we only know a limited number of values of x' and y'. Make $x'' = \frac{1}{x'}$ and construct the points whose coordinates are x'' and y'. Draw a smooth curve through them and the origin, since when $x' = \infty$, $a'' = 0$, and $y' = 0$, and we shall have between 1 and 0 a curve corresponding to any values of x' between 1 and ∞. From this we can obtain approximate values of y' for any value of x' however great. Again, let A represent the area included between the axis of Y and any ordinate y'. Now construct a curve in which $y'' =$ A, and $x'' = \frac{1}{x'}$ as before.

*This curve is not given in the original memoir of Fresnel, but being found among his manuscripts has recently been published. *Œuvres complètes d'Augustin Fresnel*, Vol. I. p. 382. The figure of this curve in Billet's *Traité d'Optique Physique*, Vol. I. Plate II. is very incorrect, F B being made equal to 1 instead of ·5

If we prolong this curve until it meets the axis of Y we obtain a value of A or the total area corresponding to $x''=0$ or $x'=\infty$, that is, the whole space included between the curve and its asymptote. Applying this method to the present case the total area of B E F was found to be ·32, ·35 and ·33, the last measure being the most reliable. The great variations are due to the fact that no values of x' greater than 5·5 could be obtained.

To determine the area of F D E B the parts above and below the line A D were compared by dividing them up into triangles, by cutting them out of sheet lead and weighing them, and by Simpson's rule applied to the ordinates of the curve, also directly to the numbers computed by Fresnel. The third and fourth methods appear most reliable, but all gave the part below the line *greater* than those above. The excess by the third method was ·355 : ·348 : ·350 : ·344 : according as the calculation was limited by the middle of an elevation or depression. The fourth method, in the same way, gave ·337 : ·334 : ·345 : ·345. Instead of an increase we therefore have a diminution of light, which is evidently due to the portion F C′ B which is inflected. Allowing for this, we have the area of the rectangle equal 11·000, that of the curved area 10·984, leaving still a deficiency of ·016, but this very small difference is easily accounted for by errors of measurement. Fresnel's numbers, moreover, do not seem to be perfectly accurate, for it will be found that the magnitudes of his maxima and minima do not diminish with perfect regularity.

Let us next see what excess of light would be needed to produce the effects observed in the photographs. A measurement of the band of light shows that its brightest part is at least one seventieth of the moon's diameter in width, or thirty miles. The unit of Fresnel in this case would be about 35 feet, hence the light received on this portion would be represented by a rectangle with base 4500 and altitude 2, or area 9000 units. If the excess then equalled the whole of the first bright diffraction band or ·6 it would only increase the brightness of the sun's disk by one fifteen thousandth. While the small difference obtained in my computation would only equal one five thousandth of one per cent. Evidently then there can be no appreciable effect due to diffraction.

Massachusetts Institute of Technology, March, 1870.

ON THE FOCAL LENGTH OF MICROSCOPIC OBJECTIVES.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.*

By Chas. R. Cross.

The investigation of which the present article is a summary, was undertaken in order to see if some reliable method of measuring the focal length of microscope objectives could not be found. The importance of such a method will be apparent to all who have had occasion to make use of objectives by different makers. The focal length of lenses of the same denomination is subject to so great a variation that comparison of these by means of their assumed focal lengths too often gives no true idea of their relative excellence. For example, if two quarter-inch objectives be compared, and one gives results much superior to that given by the other, we cannot be at all sure that the better lens is not really of shorter focus than its designation would indicate.

The question immediately arises, what is the focal length of a compound objective? The focal length of a simple lens, or of a system of lenses in actual contact, is the distance from the optical centre of that lens or system to its principal focus. But as a system of lenses not in contact, like the triplet objective, has no optical centre, the term is only a general appellation serving to group together objectives of approximately the same magnifying power. If every system of lenses possessed an optical centre, or could be replaced by a single lens, we might define the focal length of such

* In a recent number of this *Journal* (March, 1870, p. 208,) attention is called to the great need that is felt of a laboratory for physical research. In 1864, President Rogers, in his plans of this Institute, proposed such a laboratory; and, during the past year, we have fitted up rooms in which our regular students verify many of the laws and measure the principal constants of physics, while more advanced pupils carry on original investigations. In developing the first part of this plan, many unexpected results have been arrived at, and, thinking that they may be of value elsewhere, I hope to make them the subject of a future paper. I think the accompanying article by Mr. Cross will show, that although we cannot always expect to have students with his skill and perseverance, yet that such a laboratory ought to furnish many additions to physical science. E. C. Pickering.

a system as being the focal length of a single lens equivalent to the system in magnifying power. But, as this is not the case, a lens replacing the system when the conjugate foci are separated from each other by a distance, d, will not replace it if the foci are separated by any other distance, d^1; and the difference in focal length of lenses replacing the system under these different circumstances varies with the internal arrangement of the system. But for any constant distance between the conjugate foci any system can be replaced by an equivalent single lens; and in order to attach a definite meaning to the term "focal length," as applied to triplet objectives, I would propose that the focal length of such an objective be understood as being the focal length of a lens replacing the system when the distance between the conjugate foci is ten inches.

The present series of experiments was begun upon the supposition that a triplet microscope objective has an optical centre, and the formulæ applied to obtain the focal length were based on this supposition, which, on further investigation, proved to be incorrect. Observations recorded in the sequel showed, however, that though not exactly correct, the formulæ offer a very close approximation to the truth for distances of about ten inches between the conjugate foci; the variation in the focal length of the lens replacing the system corresponding to a difference of several inches in that distance being very small.

We may, then, deduce a formula by which to find an equivalent simple lens from the relations between the principal focus, the conjugate foci, and the relative magnitude of object and image for given distances between them.

The principal focus of a simple lens can be determined from the formulæ, $\frac{}{p}+\frac{1}{p^1}=\frac{1}{f}$, in which p and p^1 are the conjugate force obtained by direct experiment, and f is the principal focus sought. Even if the triple objective had an optical centre, this formulæ could not be applied directly, owing to the practical impossibility of finding this centre, which, moreover, would change with the relative change of position of the lenses composing it in varying the adjustment for different thicknesses of the glass covering the object. The direct use of the formulæ would require the distances p and p^1 to be known. Some modification must, therefore, be adopted.

The method ordinarily in use by the makers seems to be to grind

the lenses with certain radii, which are assumed to give approximately definite "focal lengths." The glasses, if tested at all, are compared with some standard objective, by means of their magnifying powers with the same eye-piece, a method liable to considerable errors.

The method which I use is based upon two equations, the one given above, $\frac{1}{p}+\frac{1}{p^1}=\frac{1}{f}$ (1) and $\frac{p}{p^1}=\frac{\text{size of object}}{\text{size of image}}=n$ (2), in which n is the ratio of the size of the object to the size of the image, the conjugate foci being p and p^1. It is clear that though we cannot measure p and p^1 separately, we can measure their sum, that is the distance between the object and the image, which we call l; n can be found by measuring the size of the image of an object of known magnitude, a finely divided scale throwing this image on a second divided scale.

We have from (1) $p^1f+pf^1=pp^1$, or $lf=pp^1=(l-p^1)p^1$ (3), as $p+p^1=l$. But $p=np^1$ from (2); therefore, $p^1+np^1=l$, and $p^1=\frac{l}{n+1}$ (4). Combining (3) and (4) $fl=\frac{nl^2}{(n+1)^2}$, and $f=\frac{nl}{(n+1)^2}$. If, therefore, we find n and l, we can easily find f, the focal length of the equivalent lens.

Two sets of experiments were made with a somewhat different arrangement of apparatus in each. In both the image of a glass scale divided to $\frac{1}{100}$ millimetres ($\frac{1}{2500}$ inch) was thrown upon an engine-divided paper scale ($\frac{1}{50}$ or $\frac{1}{60}$ inch), the image and the paper being in the focus of a double-convex lens used as an eye-piece, so that the size of the magnified image was read directly on the paper scale, estimating by the eye to tenths of the divisions. The distance from the glass to the paper was measured with a steel rule graduated to millimetres. The magnitude of the image varies so slowly for any variation of l that this was taken only in whole millimetres. Any error in the measurement would be less perceptible in the result, the shorter the focus of the lens measured.

The first form of apparatus consists of a stand with a vertical block pierced with two holes, in one of which is placed the objective to be measured with its optical axis horizontal. Through the other hole (the lower one) slides a horizontal bar, at one end of which is the micrometer used as the object, at the other end the paper scale on which the size of the image is measured, reading with an eye-piece detached from the instrument. This bar is

moved in either direction until the image is thrown directly on the paper scale, the motion being accomplished by a screw giving a fine adjustment, while the nut in which the screw plays is moved by the hand for the coarse adjustment. The length, l, between the two scales was measured once for all.

The results given by this arrangement were very satisfactory, but owing to its horizontal position and a difficulty in centering the lenses, I afterwards adopted a somewhat different apparatus, with which most of the recorded observations were made. Here I employ a microscope stand, using simply the tube and stage, placing the objective in its usual position, and throwing the image of the micrometer upon the paper scale. The latter is now gummed to the end of the tube usually occupied by the eye-piece, which is replaced by a convex lens detached from the instrument as before. The distance, l, now a variable, has to be measured with each objective. This arrangement can evidently be applied to all microscopes, rendering it possible for any one to determine the focal length of his own objectives with the greatest ease.

The following measurements will show the accuracy of the method:—

Tolles, Second Quality ¼.				Smith and Beck, ¼.			
Apparatus No. 1.		Apparatus No. 2.		Apparatus No. 1.		Apparatus No. 2.	
l = 311 mm.		l = 246 mm.		l = 311 mm.		l = 253 mm.	
f.	N.	f.	N.	f.	N.	f.	N.
·2416	20	·2424	80	·2115	20	·2108	40
·2416	10	·2419	50	·2112	30	·2097	40
·2416	20	·2415	50	·2107	30	·2097	40
·2397	20	·2415	40	·2107	30		
M = ·2411		M. = ·2418		·2107	30	M. = ·2099	
P. E. = ·00029		P. E. = ·00011		·2107	30	P. E. = ·00011	
p. e. = ·00058		p. e. = ·00022		·2104	25	p. e. = ·00019	
				·2086	20		
				·2086	20		
				·2079	20		
				M. = ·2095			
				P. E. = ·00025			
				p. e. = ·00084			

The preceding measurements were made with both forms of apparatus, and at an early period of the investigation; the following

re by the second method alone, and were taken after considerable ractice in using the apparatus.

Hartnack, No. 9.				Tolles, $\frac{1}{12}$.				Nachet, No. 7.			
Adjusted for				Adjusted for				Adjusted for			
Object Covered.		Object Uncovered.		Object Covered.		Object Uncovered.		Object Covered.		Object Uncovered.	
l = 248 mm.		l = 249 mm.		l = 257 mm.		l = 258 mm.		l = 265 mm.		l = 265 mm.	
f.	N.	f.	N.	f.	N.	f.	N.	f.	N.	f.	N.
·0819	10	·0917	10	·0640	10	·0900	15	·0721	10	·0898	15
·0816	10	·0911	10	·0636	10	·0898	15	·0721	10	·0898	15
·0816	10	·0910	10	·0636	10	·0898	15	·0718	10	·0897	10
M. = ·0817		·0910	15	M. = ·0638		M. = ·0899		·0718	10	·0896	15
P. E. = ·00006		M. = ·0912		P. E. = ·00001		P. E. = ·00004		M. = ·0719		M. = ·0897	
p. e. = ·00010		P. E. = ·00009		p. e. = ·00002		p. e. = ·00007		P. E. = ·00005		P. E. = ·00008	
l. = 249 mm.		p. e. = ·00019						p. e. = ·00010		p. e. = ·00006	
f.	N.	l = 264 mm.									
·0817	10	f.	N.								
·0817	10	·0923	10								
·0817	10	·0914	10								
·0817	10	·0911	10								
·0816	10	M. = ·0915									
M. = ·0817		P. E. = ·00020									
P. E. = ·00002		p. e. = ·00035									
p. e. = ·00006											

In the two preceding tables, the focus (f) is given in decimals f an inch, as computed from each reading. N is the number of

spaces observed on the micrometer scale, and l the distance between the two scales. The most probable mean (M) is computed by giving weight to each observation proportional to the number of spaces (N) taken; that is, assuming that the whole error lies in reading the size of the magnified image on the paper scale. P. E. is the probable error of the mean (M) and *p. e.* that of one observation.

In all the measurements, the objective was refocussed for each reading, and in those given above it was dismounted and remounted for each set.

The table below gives the results of several hundred measurements on various objectives. The first column gives the order in the table for convenience of reference; the second the name of the maker and designation as given by him; the column headed A gives the adjustment, whether for covered or uncovered glass, with the position of the index, if this is present; also whether the lens is wet or dry if an immersion lens. The next column gives in millimetres the distance denoted in the formula by l; that is, the distance between the two scales. The value of f is in all cases the most probable mean of a number of observations. The column headed B gives the focal length indicated by the maker in decimals of an inch. The next column, headed "diff. A.," gives the difference between the actual focal length, as determined by these observations, and the designated focal length as given in column B. The last column, headed "diff. B.," gives the difference in decimals of an inch between the *extreme* values of f as given from different observations by this method.

The micrometer used as an object, was a glass ruled scale divided to $\frac{1}{100}$ mm. ($\frac{1}{2500}$ inch,) except in the objectives numbered 9, 10, 16, 18, 28, 31, 32 (marked with an asterisk) in which a $\frac{1}{1000}$ in. stage micrometer was substituted, as the more finely divided scale could not be read with such low powers as these objectives gave.

Objectives Nos. 16 and 28 are single lenses; the rest are triplets. Lens No. 32, by Zentmayer, had been slightly altered in focal length to adapt it to a gas microscope. The adjustment for covered object shortens the focus of all the glasses examined, except Nos. 11, 12 and 17, in which a lengthening of focus takes place. No. 11 is adjusted for glass covering when the index is at O, contrary to the usual method in which the greater the number of the index the thicker the covering for which the objective is adjusted. Objective No 12 is No. 11 with the front lens removed and an "immersion front," a lens of different focus screwed on in its place.

No.	Name of Maker.	A	l	Focal Length.		B	diff. A.	diff. B.
			mm.	mm.	in.	in.	in.	in.
1 a	Hartnack, No 10.	Unc. Dry.	249	1·985	·0782			·0006
b		Cov'd Dry.	249	1·696	·0668			·0009
c		Unc. Wet.	249	1·991	·0784			·0002
d		Cov'd Wet.	249	1·681	·0662			·0002
2	Hartnack, No. 9.	Unc. Dry.	249	2·817	·0912			·0007
		Unc. Dry.	264	2·825	·0915			·0012
		Cov'd Dry.	248	2·079	·0818			·0003
		Cov'd Dry.	249	2·078	·0816			·0001
		Cov'd Dry.	261	2·079	·0817			·0006
		Unc. Wet.	261	2·889	·0921			·0007
3	Hartnack, ¼ in.		249	6·295	·2478	·2500	— ·0022	·0018
4	Hartnack, ½ in.		259	13·659	·5387	·5000	+ ·0387	·0027
5 a	Nachet, No. 7.	Unc'd.	265	2·279	·0897			·0002
b		Cov'd.	265	1·827	·0719			·0008
6	Nachet, No. 5.		264	3·066	·1207			·0004
7	Nachet, No. 3.		285	4·666	·1758			·0004
8	Nachet, No. 2.		239	6·469	·2547			·0006
9*	Nachet, No. 1.		244	14·791	·5823			·0012
10*	Nachet, No. 0.		268	39·108	1·5895			·0027
11 a	Ross' $\frac{1}{12}$ in.	0	249	1·675	·0659			·0004
b		22	250	1·900	·0748	·0833	— ·0085	·0018
12 a	Ross' $\frac{1''}{12}$ in., with Immersion Front by Tolles.	0 Dry.	249	1·265	·0498			·0001
b		0 Wet.	249	1·278	·0501			·0006
c		81 Dry.	250	1·696	·0668			·0005
d		81 Wet.	250	1·689	·0665			·0006
13	Smith and Beck, $\frac{1}{5}$ in.		255	4·906	·1931	·2000	— ·0069	·0017
14 a	Smith and Beck, ¼ in.	0	253	5·332	·2099	·2500	— ·0401	·0006
b		18	253	5·038	·1983			·0005
15	Smith and Beck, ⅔ in.		257	16·330	·6429	·6667	— ·0238	·0000
16*	Smith and Beck, 1½ in.	Single Lens.	276	33·990	1·3382	1·5000	— ·1617	·0038

No.	Name of Maker.	A	l	Focal Length.		B	diff. A.	diff. B.
			mm.	mm.	in.	in.	in.	in.
17 a	Collins, $\frac{4}{10}$ in.	3	258	8·180	·8220			·0009
b		20	259	8·502	·8347	·4000	— ·0653	·0010
18*	Collins, $1\frac{1}{2}$ in.		271	42·572	1·6761	1·5000	+ ·1761	.0047
19 a	Spenser, $\frac{1}{2}$ in.	Unc'd.	266	9·717	·3826	·5000	— ·1174	·0000
b		Cov'd.	266	8 947	·3522			·0018
20 a	Tolles, "$\frac{1}{12}$ in. dry, $\frac{1}{15}$ in. wet."	0 Dry.	258	2 286	·0899	·0833	+ ·0066	·0002
b		19 Dry.	257	1·620	·0638			·0004
c		6·3. Dry.	256	2 024	·0797			·0012
d		6·3. Wet.	256	1·995	·0786	·0667	+ ·0119	·0003
21	Tolles, $\frac{1}{12}$ in.	0 Dry.	257	2·213	·0871	·0833	+ ·0038	·0011
22	Tolles, $\frac{1}{10}$ in.	0 Dry.	257	2·898	·1141	·1000	+ ·0141	·0014
23 a	Tolles, $\frac{1}{10}$ in.	0 Dry.	258	2·874	·1131	·1000	+ ·0131	·0000
b		14 Dry.	259	2·283	·0899			·0008
c		0 Wet.	258	2·910	·1146	·1000	+ ·0146	·0005
d		14 Wet.	259	2·263	·0891			·0004
24 a	Tolles, $\frac{1}{8}$ in.	0	258	3·217	·1267	·1250	+ ·0017	·0011
b		$16\frac{1}{2}$	258	2·683	·1056			·0005
25 a	Tolles, $\frac{1}{4}$ in.	0	263	6·083	·2395	·2500	— ·0105	·0011
b		10	263	5·675	·2234			·0026
26	Tolles, $\frac{1}{4}$ in., 2d Q.		246	6·143	·2418	·2500	— ·0082	·0009
27	Tolles, $\frac{1}{4}$ in., 2d Q.		248	6·400	·2520	·2500	+ ·0020	·0010
28*	Tolles, 1 in., 2d Q.	Single Lens.	259	25·188	·9916	1·0000	— ·0084	·0000
29 a	Wales, $\frac{4}{10}$ in.	0	258	6·818	·2684	·4000	— ·1316	·0008
b		9	258	6·563	·2584			·0007
30	Zentmayer, $\frac{4}{10}$ in.		255	9·193	·3619	·4000	— ·0381	·0009
31*	Zentmayer, $\frac{8}{10}$ in.		269	20·113	·7918	·8000	— ·0082	·0000
32*	Zentmayer, $1\frac{1}{2}$ in.		280	36 839	1·4504	1·5000	— ·0496	·0022
33 a	Grunow, $\frac{1}{4}$ in.	Unc'd.	273	5·253	·2068	·2500	— ·0432	·0012
b		Cov'd.	273	5·059	·1992			·0008
34	Grunow, $\frac{1}{2}$ in.		255	8·902	·3505	·5000	— ·1495	·0021

The contents of column A may need some further explanation. The immersion lenses were measured both wet and dry to determine the change of focal distance, which is very small unless the interior arrangement of the lenses is altered by moving the index circle. The figures in this column are the numbers given by the index if there was one; if no index was attached to the objective the extremes of the adjustment were taken, and are indicated by the words "Unc." for uncovered, and "Cov'd" for covered adjustment. The $\frac{1}{100}$ millimetre scale used as the object was always uncovered, whether the lens was adjusted for covered and uncovered objects, as no other method seemed so generally applicable. This, of course, rendered the definition somewhat indistinct when the lens was adjusted for glass covering, which will explain the greater difference in the corresponding extreme values of f in the last column.

The measurements in the preceding table were made with the second form of apparatus, so that the length of l varies slightly from the normal length of 10 inches. The extreme values are 276 mm. (10·87 inches) for No. 16 and 235 mm. (9·25 inches) for No. 7. To ascertain the effect of this variation on f, the computed focal distance, the following observations were made. 1st. A Smith and Beck $\frac{1}{4}$ inch (No. 14) was measured, first with $l = 279$ mm. (10·98 inches), and then with $l = 412$ mm. (16·22 inches). The computed values of f in the two cases were—

$$l = 279 \text{ mm.}, f = \cdot 2102 \text{ in.}; \quad l = 412 \text{ mm.}, f = \cdot 2035 \text{ in.},$$

giving a difference of only ·0067 inch in f for a difference of 133 mm. (5·24 inches) in l. 2d. A Tolles second quality $\frac{1}{4}$ inch (No. 26) was measured in the same way, giving values of f as follows:

$$l = 259 \text{ mm.}, f = \cdot 2424 \text{ in.}; \quad l = 414 \text{ mm.}, f = \cdot 2395 \text{ in.}$$

a difference of but ·0029 inches, corresponding to a difference of 155 m m. (6·10 inches) in l. From these results it was inferred that with the maximum deviation (in No. 16) of 22 m m., (0·87 in.) from the normal value of l, the correction required to reduce the value of f to that standard length, would be within the limits of probable error, and in most of the objectives the deviation of l is far less than in this case.

An examination of the table will show that the focal length of the objectives of some makers differs considerably from the length marked upon them. For example, No. 34 marked $\frac{1}{2}$ inch is really

a $\frac{1}{8}$ inch objective; No. 33 marked $\frac{1}{4}$ inch is really a $\frac{1}{5}$ inch; No. 29 marked $\frac{4}{10}$ inch is a $\frac{1}{4}$. Lens No. 14 marked $\frac{1}{4}$ inch is really a $\frac{1}{5}$ inch; but Nos. 13, 15, by the same makers, are correctly designated $\frac{1}{5}$ inch, $\frac{2}{3}$ inch. Differences of this kind must of necessity lead to a great confusion in comparing objectives with one another. I would therefore suggest that each objective made should be measured before being offered for sale, that this confusion may cease to exist. A convenient arrangement would be to fix a glass scale divided to $\frac{1}{50}$ or $\frac{1}{100}$ inch in the draw-tube, sliding in the tube of the microscope, and measure as I have already described. The draw-tube should be moved till the front of the ruled glass shall be exactly 10 inches from the micrometer used as the object. Or it would be more convenient still to have an apparatus similar to the first form, but arranged with a suitable stage and stand so that it can be set at any desired angle. The distance 10 inches (254 mm.), suggested as a standard is chosen because it is the normal distance of distinct vision, as well as about the length used by microscopists in actual work.

An inspection of the formula $f = \frac{n\,l}{(n+1)^2}$ shows (1) that the focal length of any lens is not inversely proportional to its magnifying power with a given distance (l) between the conjugate foci, as is commonly assumed, but to $\frac{n}{(n+1)^2}$, the ordinary supposition approaching absolute correctness as n increases. Hence the inaccuracy of any system of estimating focal lengths upon this assumption when applied to lenses of long focus.

(2.) The shorter the focal length of the objective, the less will any error in the measurement of l affect the result.

(3.) Any error in the measurement of n also affects the result less in a lens of short focus. It would therefore appear that by this method the most accurate results are obtained with the objectives of highest power. The following examples from the records of my observations will illustrate this last point. The numerators of the fractions are the readings on the paper scale, the denominators, the number of spaces of the micrometer scale corresponding to these readings, the quotient being of course the value of one of these magnified spaces in fiftieths of an inch.

With objective No. 16*, Smith and Beck, 1½ inch—

Scale readings $\frac{32 \cdot 8}{11} = 2 \cdot 982$. $\frac{32 \cdot 7}{11} = 2 \cdot 973$. $\frac{32 \cdot 7}{11} = 2 \cdot 973$.

Give values of $n = 5 \cdot 964$. $5 \cdot 946$. $5 \cdot 946$.

The corresponding values of f are—

1·3362 inch. 1·3392 inch. 1·3392 inch.

$l = 276$ mm. $M = 1 \cdot 3382$ in. Differences of extremes $= \cdot 0030$ in.

With objective No. 8, Nachet No. 2.

Scale readings $\frac{41 \cdot 2}{60} = \cdot 6866$. $\frac{41 \cdot 2}{60} = \cdot 6866$. $\frac{34 \cdot 4}{60} = \cdot 6880$. $\frac{41 \cdot 3}{60} = \cdot 6883$

Give values of $n = 34 \cdot 879$. $34 \cdot 879$. $34 \cdot 950$. $34 \cdot 965$.

The corresponding values of f are—

·25494 inch. ·25494 inch. ·25446 inch. ·25435 inch.

$l = 239$ mm. $M = \cdot 25466$ in. Difference of extremes $= \cdot 00059$ in.

With objective No. 1, *c*, Hartnack, No. 10.

Scale readings $\frac{36 \cdot 4}{15} = 2 \cdot 447$. $\frac{24 \cdot 2}{10} = 2 \cdot 420$. $\frac{36 \cdot 3}{15} = 2 \cdot 420$. $\frac{36 \cdot 3}{15} = 2 \cdot 420$.

Give values of $n = 123 \cdot 29$. $122 \cdot 94$. $122 \cdot 94$. $122 \cdot 94$.

The corresponding values of f are—

·07824 inch. ·07845 inch. ·07845 inch. ·07845 inch.

$l = 249$ mm. $M = \cdot 07839$ in. Difference of extremes $= \cdot 00021$ in.

The increasing change in f for the same variation of the scale reading is clearly seen on comparing the above sets of observations. The diminution of the number of divisions measured in a short focus objective is a partially neutralizing circumstance, which can, however, be avoided by using a lens of long focus for the eye-piece, so as to gain a larger field of view.

The chief difficulty met with in pursuing this research was that of procuring a suitable scale for the object, the image of which was to be meaured. In the earliest measurements a scale on glass ruled to $\frac{1}{1000}$ inches was used, but the lines were jagged at the edges, their breadth was variable and their spacing unequal. Next an eye-piece micrometer belonging to a Smith and Beck's microscope was used, the divisions being $\frac{1}{250}$ inch, but though this was an improvement on the former the results were still unsatisfactory. Finally a micrometer reading to $\frac{1}{100}$ millimetres was used, which was all that could be desired in clearness and evenness of lines and equality of spacing. In some cases, for long focus objectives a $\frac{1}{1000}$ inch micrometer scale was substituted, as before stated. The paper scale used to measure the size of the image was divided to $\frac{1}{50}$ or $\frac{1}{80}$ of an

inch, while the
giving the mea
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Boston, May 3d,

[*From the Journal of the Franklin Institute.*]

THE GRAPHICAL METHOD.

By Prof. Edward C. Pickering.

One of the most valuable means of studying physical laws is the graphical method, or the representation of phenomena by curves. If any two quantities are so connected that an alteration of one produces a change in the other, a curve may be constructed in which ordinates and abscissas correspond to the magnitudes of these variables. The principal objection to this method is its inaccuracy, and the object of the present article is to show how this difficulty may be avoided, and the errors reduced to any desired magnitude. One of the most accurate applications of the graphical method was that of Regnault, in his study of the laws of heat. He used a copper plate, ·8 of a metre square, and constructed points by means of a miniature dividing engine, giving hundredths of a millimetre, or five places of decimals. Probably, however, the last of these would be very doubtful, and in the printed sheet even the fourth figure is liable to an error of one or two units. Again, it is difficult to draw a curve through a series of points, unless they fall near together, and therefore, in practice, we can hardly depend on more than three places of figures with certainty. That is, if all our points fall between 0 and 1, our errors should not exceed one-thousandth, if between 0 and 100, one-tenth.

The most obvious means of diminishing errors is to enlarge the scale. This is, however, limited by the size of the paper, and has moreover the disadvantage of making the points fall further apart, and thus rendering it more difficult to draw a smooth curve through them. In many cases, therefore, a large curve is no better than a small one. This objection does not apply to the straight line and circle, since they may be constructed by the ruler and compasses, but it then becomes necessary to construct all the points to a scale, as a distortion is almost always introduced in paper ruled or engraved in squares, by its unequal expansion by moisture. The next question is the degree of accuracy as affected by the inclination of the curve to either axis. Generally, after constructing our curve, we

wish to know the magnitude of one variable corresponding to certain values of the other, that is, we draw horizontal lines corresponding to certain assumed values of y, and the abscissas of the points where they meet the curve will give the corresponding values of x. If the curve is nearly horizontal, large errors may be introduced, owing to the obliquity of the intersection, and if nearly vertical, there is the same trouble with the values of y. Now, it is by no means necessary that abscissas and ordinates should be taken to the same scale; in fact, they usually represent different units. By increasing the scale on which abscissas are constructed, we render the curve more nearly horizontal, increasing the scale of ordinates, more nearly vertical. The question then arises with what degree of accuracy can these intersections be found at different inclinations, and what scales must be adopted to give the best results. Evidently, the error will be proportional to the space through which the lines coincide, or to their thickness divided by the sine of their angle of intersection. Calling, therefore, α the angle the curve makes with the axis of X, the error in determining $x = \frac{e}{\sin. \alpha}$, and for $y = \frac{e}{\cos. \alpha}$, e being the error when the lines are as right angles, or its minimum value. When $\alpha = 45°$, the two last errors $= 1{\cdot}4\, e$, the most favorable case. When $\alpha = 30°$, the error of x or $e_x = 2\,e$ „ $e_y = 1{\cdot}12\, e$; $\alpha = 90°$, gives $e_x = 0$; but $e_y = \infty$, or the point of intersection cannot be obtained since the two lines coincide. One conclusion then is that the scale must be taken such that the curve will not be very oblique to either axis, the best effect being attained with angles of 45°. When the scale is enlarged in one direction only, the accuracy is not proportional to the enlargement, but depends on the direction of the curve. As the direction of a curve is commonly given by the tangent of the angle it makes with the axis of X, let tang. $\alpha = \frac{a}{b}$: then $e_x = \frac{\sqrt{a^2 + b^2}}{b} e$; $e_y = \frac{\sqrt{a^2 + b^2}}{a} e$. If we enlarge the scale of x, m times that of y, n times, we have $e_x = \frac{\sqrt{a^2\, m^2 + b^2\, n^2}}{b\, m} e$.

$e_y = \frac{\sqrt{a^2\, m^2 + b^2\, n}}{a\, n} e$, from which we can readily compute the ased accuracy due to the enlargement in any case.

us now suppose that we have a sheet of paper 1 metre square

divided into millimetres, and that on this we have constructed a curve to such a scale that it extends nearly diagonally across the sheet, and that all our observed points agree with theory within a millimetre. It will generally be only difficult to decide whether these errors, although occurring in the fourth place of decimals are real variations from theory or only accidental errors. Moreover, only a very small part of the paper is used, the ruling on ninety-nine hundredths of it being quite useless. We then construct a curve in which while x is unchanged, y represents the deviation between the observed and theoretical curve the scale being enlarged 10 or 100 times. Evidently the errors will now become so large that we can tell at a glance whether they are accidental or constant, and if the latter, how the curve must be altered to diminish them, or by what amount we must correct any point of our theoretical curve to make it agree with observation.

This method has special value in obtaining empirical formulas from series of observations. We assume some simple equation as a first approximation, and construct points as before, giving differences on an enlarged scale. Treat this new curve precisely like the first one, assuming a second approximate equation, and so proceed until all deviations except those of observation are eliminated. Placing y equal to the sum of all these assumed values, (first reducing them all to the same scale) we have the required empirical equation. To put the matter in a mathematical form, let x'' and y'' be co-ordinates of each observed point in succession. Assume the curve $y = f(n)$ which shall nearly coincide with them and construct the points whose coördinates are x'' and $[y'' - f(x'')]\ 10$. As an approximation to this last curve take the formula $y = f'(x)$, and construct again a curve with co-ordinates x'', and $\left([y'' - f(x'')]\ 10 - f'(x'')\right) 10$. Having finally destroyed the constant errors we have the required equation $y = f(x) + \frac{f'(x)}{10} +$ &c. It is best where practicable to assume some simple value of $f(x)$, $f'(x)$, as $a\,x + b$ or $a \log x + b$ making $a = 1, 2, 3, \cdot 5,$ &c. Much labor is thus saved and we can often find a simple equation which will satisfy observation as well as any complex one. The examples given below explain this method more perfectly than any description, and show how easy it is to compare different empirical formulas.

A point of inflexion is readily found by assuming equations of the form $f(x) = a\,x + b$, and making these lines approximately tangent to the curve at the required point. Maximum and minimum values of y are found by drawing the axis of X very near these points and enlarging y.

Asymptotes present especial difficulties to the graphical method as commonly used. Suppose we have a curve asymptotic to the axis of X. Construct a curve in which while y is unchanged x shall be the reciprocal of that previously taken. Then all points of the curve between 1 and ∞ will now be included between 1 and 0. It is often desirable to determine the area included between the curve and asymptote. If this is finite our new curve will be tangent to the axis of x at the origin. Its magnitude may be determined by constructing a third curve in which x is, as before, the reciprocal of its first value, while y for any point is proportional to the area included between its ordinate and that of some other point assumed as an origin. These values of y may be obtained from the original curve by the usual processes of measurement. The ordinate of the point where our new curve meets the axis of y gives the total area required. As an application of this device, see an article by the writer in this *Journal*, March, 1870, entitled "Diffraction along the Moon's Limb."

Of course all these methods would only be used where the application of the calculus is impossible. The true test of the excellence of the devices here proposed is to apply them to some known series of observations, and for this purpose I have selected those of Regnault on the latent heat of steam, its pressure, and on the absolute dilatation of mercury.

I. *Latent Heat of Steam.*—Four series of experiments were made, and from them he concluded that the total heat was best represented by the formula $T = 606{\cdot}5 + {\cdot}305t^{\circ}$. I therefore assumed this as a first approximation, and constructed points as in Plate I. in which temperatures are measured horizontally and the total heat vertically, the unit of the latter being ten times that of the former.

that ries I. is confined to determinations very near 100°. Six pre-
xperiments give the points represented by crosses. The
$= \sqrt{a}$ contained between 638·3 and 635·5 or + 1·3 and — 1·5.
hows the mean of the latter which is 636·73, the circle
ased accu e whole 44, or 636·35. The probable errors of these
us now s nd ·13. The formula giving 637 is evidently too

Series II. relates to temperatures between 100° and 200°. The formula again gives too large results.

Series III. gives the measurements between 50° and 100°. With a single exception every point is below the axis.

Series IV. for low temperatures (near 0°) gives very scattering results. They are, however, in general above the axis.

If we simplify the formula of Regnault by substituting ·3 for ·305, so that it shall read $\text{T} = 606{\cdot}5 + {\cdot}3t$, we obtain the line A C, which agrees much better with the observations of series I., II. and III., and nearly as well with series IV. Its agreement with series I. is all that could be desired, being less than the mean, if we reject the first six observations, and greater than it, if we retain them. Giving them a weight of ·7 makes the mean coincide precisely with our formula. To show with greater certainty the advantage gained by the change here proposed, I have computed the probable errors regarding the old and new formulas successively as correct.

Series.	I.	I.	II.	III.	IV.	All.
Number observations	38	44	73	23	22	156
Probable error, Regnault's formula	·55	·87	1·27	2·00	3·15	1·72
Probable error, proposed formula	·53	·74	1·07	1·77	3·15	1·60

The probable errors according to the new formula being evidently less than the others we conclude that the total heat of steam may be expressed by the formula:—

$$\text{T} = 606{\cdot}5 + {\cdot}3\ t,$$

and that the total heat at the boiling point is 636·5 instead of 637 as commonly taken.

II. *Absolute Dilatation of Mercury.*—Regnault gives as the best formula for the absolute increase of volume of mercury by heat. $\text{I} = {\cdot}0001790\ t + {\cdot}000002523\ t^2$, in which I is the *increase* of volume and t the temperature. I first assumed the simple equation $\text{I} = {\cdot}00018\ t$, and constructed differences to a scale 176 times that of Regnault, (A A, Pl. II.) The points followed approximately a parabola with vertex at the origin, showing that it was necessary to take into account the second power of t. My second approximation therefore was ${\cdot}000002\ t^2$, which gives a result represented in Plate II. I was now

enabled to enlarge 10 times more than before, or 1760 times, which corresponds to a unit about a mile in length. That is, if one column of mercury was one mile long its change in volume would be shown in its true dimensions. The points of each series of observations are connected together giving four zigzag lines for the four series. The curved line B B shows the values computed by Regnault's formula. Series I. is evidently better satisfied by the new formula as every point falls above the curved line. Series II., on the other hand, agrees best with the formula of Regnault. Series III. and IV. agree much better with the axis except for temperatures above 270°. Although there is much uncertainty attending this measurement, yet there seems for these points a decided tendency to rise above the axis.

Computing as before the probable errors, we have:—

Series.	1	2	3	4	All.
Number of observation	4	6	11	14	35
Regnault's formula	3·4	3 8	5·4	5·6	5·1
Proposed formula	2 2	4·5	3·7	5·0	4·3

The unit is ·00001. We see then that every series except the second gives a less probable error, while the much greater ease of computing with, or remembering the new formula is obvious if we write one below the other thus:—

Regnault's formula, $I = \cdot 0001790\ t + \cdot 000002523\ t^2$.
Proposed formula, $I = \cdot 00018\ t \quad + \cdot 000002\ t^2$

If we take our unit of temperature 100° instead of 1° we have $I = \cdot 018\ t + \cdot 02\ t^2$.

III. *Pressure of Steam.*—The principal object of Regnault's researches was to determine this law. He drew a smooth curve through his observed points, and then compared it with several empirical formulas. As he gives the differences in numbers it is difficult to compare them with one another. I have therefore constructed points in Pl. III. in which the horizontal distances represent temperatures, vertical distances, the difference in pressure computed by the formula and that given by the curve. The unit is 1 mm., and the scale below 100° ten times that above.

The exponential formula of Biot was first tried. Log. $F = a + b\alpha^x + c\beta^x$, in which F is the pressure, x the temperature plus a constant. In equation (F) which relates to temperatures above 100°, $x = T - 100$, in (H) $x = T + 20$, and applies to all temperatures. M. Roche proposed the formula $F = a\,\alpha^{\frac{x}{1+mx}}$ founded on theoretical considerations and represented by K. The deviation here being greater, the exponential formula was adopted, and by it the common table for the pressure of steam was computed. It will be noticed that certain irregularities are common to all, for instance, at 150° and 200°. These were evidently due to the impossibility of constructing graphically a perfectly smooth curve. As, moreover, the deviation in many cases amounts to one centimetre we see that even the third place of decimals is sometimes doubtful.

The above examples show that it is perfectly possible thus to render obvious the errors even in the most accurate series of experiments, while from a curve we are able to judge with far more certainty of the nature of the errors and the best means of diminishing them, than it is possible to do from the numerical results.

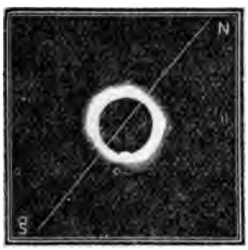
Mass. Inst. of Technology, Nov. 1st, 1870.

[*From the Journal of the Franklin Institute.*]

PHOTOGRAPHING THE CORONA.

By Prof. EDWARD C. PICKERING.

THE difficulty in photographing the corona visible around the sun during a total eclipse, is mainly due to its small actinic power. To remedy this we must increase the light in our camera as much as possible, and therefore when attached to Prof. Morton's Eclipse Party, in August, 1869, I proposed that a common portrait camera should be used. As with such an instrument we can obtain an impression of objects in a comparatively dark room in a few seconds, it seemed probable that in two or three minutes so bright a body as the corona would produce a very distinct impression, even of its more remote portions. We found in Mt. Pleasant where we were stationed, two photographers, Messrs. Hoover Bros., who undertook to give this plan a trial, and as a result they obtained a photograph, which is represented in the accompanying figure to double its original size. I believe the exposure lasted during nearly the whole period of totality, the apparent motion of the sun being avoided by following it with the camera. The aperture of the lens being much greater compared with its focal length than that of any telescope, so much light is concentrated that an impression of a large part of the corona is obtained, giving one of the best photographs of this body yet taken.



A comparison with the view taken by Mr. Whipple in Shelbyville shows many points of resemblance, and greatly strengthens any conclusions based on either. It also proves that the structure common to both is solar, or at least not due to any local irregularities in our own atmosphere. The indentation in the moon's limb marks the position of the large protuberance then visible, and we readily perceive the bases of the five points or streamers which were noticed at the same time. The line N. S. gives the direction of the sun's polar axis and shows the increased height of the corona at its equator, and the corresponding diminution at its poles. The experiment is so easily tried by any photographer on the line of totality as to encourage the hope that in future eclipses, views may be taken from a great many points with the largest portrait cameras, and thus eliminating all local effects, show with certainty how much of the corona is really solar.

II. OPTICS.

1. On Dispersion, and the Possibility of Attaining Perfect Achromatism. By Edward C. Pickering, of Boston, Mass.

When spectra are obtained with prisms of different materials, we find that the colors are unequally dispersed, one giving greater prominence to the red, another to the blue. Accordingly if we attempt to neutralize the effect of one by another, or to obtain achromatism, we always find that a certain amount of color remains, forming the residuary or secondary spectrum. Let us see what are the conditions, in order that this may disappear. Let α, α' be the angles of the two prisms, n, n' the indices of refraction for any ray, and D, D' the corresponding deviations. When the angles are small we have

$$D = (n - 1)\,\alpha,\ D' = (n' - 1)\,\alpha',$$

or the deviation of both prisms

$$D'' = D - D' = (n - 1)\,\alpha - (n' - 1)\,\alpha'.$$

Now Cauchy has shown that the index of refraction of a ray of wave-length λ is represented by the formula

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \&c.$$

Substituting in the above formula, we have,

$$D'' = \left(A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \&c. - 1\right)\alpha - \left(A' + \frac{B'}{\lambda^2} + \frac{C'}{\lambda^4} + \&c. - 1\right)\alpha'$$

$$= (A - 1)\,\alpha - (A' - 1)\,\alpha' + \frac{B\alpha - B'\alpha'}{\lambda^2} + \frac{C\alpha - C'\alpha'}{\lambda^4} + \&c.,$$

and our condition is, that this shall be the same for all values of λ. By the rule for simultaneous equations all the co-efficients of λ must equal zero, or

$$B\alpha - B'\alpha' = 0,\ C\alpha - C'\alpha' = 0 \text{ or } \frac{\alpha}{\alpha'} = \frac{B'}{B} = \frac{C'}{C}.$$

The recent researches of Van der Willingen and others show that the succeeding terms may be neglected. Our last equation may be written in the form $\frac{B}{C} = \frac{B'}{C'}$; and we therefore see that, if we can find two substances in which this ratio shall be the same, by combining them we shall obtain perfect achromatism, provided the

Hilgard, I am permitted to submit, through him, to the inspection of the Association.

This iron trough has a circular plateau of about six inches in diameter. The deepened margin around the plateau increases the diameter to six and one-half inches. This deepened margin, at its outer boundary, is extended downward all around in a very narrow annular passage to the depth of three-fourths of an inch below the level of the plateau, where it opens horizontally outwards into an annular reservoir, one-half inch wide, which surrounds the plateau, and rises a fraction of an inch above it. This annular reservoir is closed air-tight above, but is provided with a screw-valve to control the passage of air. When this screw-valve is opened, and air forced in through a flexible tube with the mouth or otherwise, the mercury in the annular reservoir is forced through the annular passage and flows over the plateau, and is sufficient in quantity to flood it throughout. The continuity of surface over the plateau having been secured, the mercury is allowed to flow back into the annular reservoir; and the whole quantity of mercury may be adjusted so as to settle to the depth that is desired on the plateau. The rapidity of this return flow of the mercury may be controlled by checking the escape of air through the screw-valve. Should any accident cause the breaking of the film of mercury, the arrangement here described affords the means of its convenient and speedy renewal. The film will never break except by accident.

The screw-valve can also be used to suspend the return flow until the trough, which is furnished with levelling-screws, can be levelled by the surface of the mercury. Three steel points brought down upon the surface afford the means at once of levelling the trough, and of measuring the thickness of the film of mercury on the plateau.

liquid as a mixture of sulphuric acid and water of exactly the right dispersion. The lens might be made of two disks of glass with the intervening space filled with the liquid. It is claimed that in this way the great defect in the achromatic lens may be remedied. Spherical aberration might be avoided mechanically, if lenses could easily be ground to ellipsoids or hyperboloids; but the removal of chromatic aberration, which is inherent in the nature of the substances used, has heretofore been generally considered impossible.

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2. On Methods of Illustrating Optical Meteorology, particularly the Formation of Halos and Coronæ, according to the Theory of Bravais. By Joseph Lovering, of Cambridge, Mass.

Optical Meteorology has been developed mathematically with greater success than any other department of this complex science. The principal features of a fully developed halo are: 1. The inner circle, concentric with the luminary, and having a radius of about 22°. 2. The outer circle, also concentric with the luminary, and having a radius of about 46°. Both of these circles, called the smaller and larger halos, are tinged with the colors of the spectrum, the blue being the outermost color. 3. The parhelion circle which passes through the luminary and is parallel to the horizon. This circle is white. 4. Upon this circle, and at a distance of 22° or more from the luminary, are two mock suns, the edge toward the sun being reddish and the opposite edge bluish. 5. A sort of tail stretching from these mock suns horizontally, and opposite to the line which connects them with the sun, to the distance of 43° 28′ or more from the sun. 6. The tangent curve to the inner halo. 7. The tangent curve to the outer halo.

All these features of the halo are satisfactorily explained by refraction and reflection, produced by hexagonal prisms of ice, floating or sinking in the higher regions of the atmosphere. These particles may be so situated as to present three independent cases. 1. They may be indiscriminately in all possible positions. 2. The axes of the prisms may be parallel and vertical, the sides of

angles of the prisms are in the ratio of B' to B. There is, however, one exception to this rule, for if

$$(A-1)\,B'-(A'-1)\,B=0 \text{ or } A=\frac{A'\,B+B'-B}{B'},$$

the value of D'' becomes 0, and there is achromatism, but no deviation.

If we attempt to apply these principles to the measurements of Dale and Gladstone, or even to those of Fraunhofer, we find that the errors of observations are so great that but little reliance can be placed on the results. We find, however, that $\frac{C}{B}$ for water, other liquids of small refrangibility, and crown-glass, is very small, and generally negative; of flint glass, it is considerable and positive; while with bisulphide of carbon, phosphorus, &c., it is positive and very large. The recent measurements of Van der Willingen afford data for more accurate calculation. In the following table the index of refraction is given by the formula

$$n=A+\frac{B(10)^2}{\lambda^2}+\frac{C(10)^6}{\lambda^4},$$

λ being given in millionths of a millimetre. The first five instances show the effect of adding sulphuric acid to water. The next three are measurements of distilled water, two by Van der Willingen, the third by Fraunhofer, and show how uncertain these constants still are. The last three relate to more refrangible substances.

Substance.	A	B	C	$\frac{C}{B}$
Distilled Water	1.323142	35.01	—48.8	—1.35
HO, SO_3 23.29	1.350940	40.70	—88.8	—2.18
" 47.22	1.380046	45.74	—123.0	—2.69
" 71.97	1.411199	49.71	—152.0	—3.06
" 94.72	1.419576	45.04	—116.1	—2.58
Distilled Water, V. d. W.	1.323228	35.30	—52.6	—1.49
" " "	1.323002	36.44	—67.6	—1.80
" " F.	1.323512	36.70	—66.1	—1.86
Flint Glass	1.714502	108.70	+665.2	+6.12
Hydrate Cinnamyl	1.575443	90.57	+2074.0	+22.90
Essence Anise	1.519745	73.68	+778.5	+10.57

If we had values of $\frac{C}{B}$ corresponding to all known transparent substances, we could at once determine which would be most suitable for a lens; it would, however, evidently be easy to select some substance as crown-glass, measure $\frac{C}{B}$ for it, and then prepare a

REPORT ON THE

PHYSICAL LABORATORY

OF THE

MASS. INSTITUTE OF TECHNOLOGY.

To PROFESSOR J. D. RUNKLE, *President pro tempore of the Institute:* —

As the method of giving instruction in Physics by means of a laboratory is now attracting a good deal of attention both in this country and in Europe, it seems desirable to make known the success which has attended a laboratory of this kind lately established at the Institute. Its history is as follows: —

1864. President Rogers[1] proposed a laboratory in which "the student may be exercised in a variety of mechanical and physical processes and experiments."

1868, Oct. A room was opened to advanced students where they carried on physical investigations, as is done by many physicists, with their special students.

1869, April. The plan described below was presented by the writer and adopted by the Government of the Institute.[2]

1869, Oct. This plan was carried out, and the laboratory opened to the regular students of the Institute.

[1] Scope and Plan of the School of Industrial Science of the Mass. Institute of Technology, p. 24.

[2] Supplement to the Fourth Annual Catalogue. Plan of the Physical Laboratory. 8vo. Pamph., pp. 16.

Its first object is to enable the regular students of the Institute, after attending a course of lectures on Physics, to verify its laws and measure its constants, also to learn to use the more important pieces of apparatus. Secondly, to instruct special students in the use of particular instruments, or branches of physics, as the spectroscope, microscope, photometry, electrical measurements, etc. Thirdly, to prepare teachers of this science. And fourthly, to afford facilities to physicists to carry on investigations at the Institute.

In carrying out the first part of this plan the following difficulties presented themselves. How could experiments be performed by twenty or thirty students at a time, without duplicating apparatus to an extent which would involve a very great outlay, and how could each obtain sufficient attention from the instructor, to enable him to work with advantage? Moreover, since physical apparatus is often delicate and expensive, would not the injury and breakage, both in moving and using it, be so great as to render the current expenses of the laboratory very large? It will be seen that these difficulties were obviated by the adoption of the following plan. Two adjacent rooms were fitted up with tables, and supplied with water and gas as in a chemical laboratory. On each table is placed the apparatus necessary for a single experiment together with a complete written description. Between the two rooms is an indicator, or frame with cards containing the names of the experiments, and opposite each, a second card with the name of a student. Each member of the class on entering the rooms notices where his card is placed, goes to the proper table, reads the description, and sometimes is able to perform the experiment without any assistance from the instructor. The time of the latter is therefore sufficiently unoccupied to enable him to go from desk to desk, and watch carefully that no serious errors are committed. When an experiment is completed the card of the student is placed opposite some unoccupied table and he goes on as before. Since the number of experiments is

greater than that of students, several places are always vacant, and delays are avoided. New experiments are added at intervals to replace those which have been performed.

By this plan there is no need of duplicating apparatus, and since it always remains in the same place, the danger of breakage is very small; in fact, during the past year the loss in this way has amounted to almost nothing. It is, moreover, always ready for use, and so much time is saved in this way, that each student accomplishes a large amount of work. So much more, in fact, than was anticipated, as to cause during the past year some difficulty in supplying enough new experiments. The nature of the work done is best shown by the following examples.

Deflection of Beams. I. A steel bar rests on two knife-edges and to it is attached a scale-pan on which are placed successively, 100, 200, 300 to 1900 grammes, and the flexure measured by a micrometer screw, reading to $\frac{1}{2000}$ of an inch. The point of contact is determined with the utmost precision by connecting micrometer and bar with a galvanic battery and galvanometer, in such a way that the current only passes when the two former are in contact. The screw is turned until the needle moves, and the reading is then taken. The results are represented by a curve, in which horizontal distances correspond to weights, and vertical to deflections. This should give a straight line, and in many cases the difference is so small as to be scarcely perceptible to the eye. The experiment is then repeated, keeping the weight constant in amount, but altering its position. By moving knife-edges, micrometer or weight, and changing the magnitude of the latter, this experiment may be infinitely varied.

Deflection of Beams. II. A mirror is attached to the end of a beam, and the image of a scale is viewed in a telescope placed opposite it; a method of very general application for measuring small changes of position.

Hook's Joint. A model of Oldham's modification of this form of universal joint, has a graduated circle attached to each axle. One is made to vary uniformly 5° at a time, and the difference in position of the two wheels is recorded. The result is represented by a curve, in which horizontal distances represent the position of the first wheel, and vertical the difference in the two readings. The values given by the formula tang a = cos A tang b, are then computed, and the theoretical curve constructed on the same sheet of paper.

Law of Lenses. The image of a gas flame is projected on a screen by means of a lens, and the latter is moved half a foot at a time.

The image is focussed by moving the screen, and the positions of the latter are compared with the theory by means of two curves, as in the preceding experiment.

Spherometer. The radius of curvature of each surface of two lenses is measured, and their foci computed.

Hydrometers. The specific gravity of several liquids is measured by these instruments, also of a solid by Nicholson's hydrometer.

Gauge Flask. The specific gravity of a liquid and solid is measured in this way also.

Mohr's Balance. The specific gravity of similar substances measured a third time by the hydrostatic balance.

Specific Heat. The specific heat of a stone measured by the calorimeter, first testing the method by water.

Opthalmoscope. A model of the eye intended for use with this instrument has been procured, together with representations of the principal diseases of the retina, and means of imitating near or far-sightedness and astigmatism. The student views the retina under these various conditions, using both the direct and the inverted image, and finally a diseased retina is inserted, and he is required to draw it as seen with the opthalmoscope.

Spectroscope. The principal lines of the solar spectrum, also those of the spectra of soda, lithium, baryta, strontia and lime, are measured and drawn. Three mixtures of these substances are then given to the pupil, which he is required to analyze.

Microscope. I. With this instrument the student views various objects, one being selected to show the effect of a diaphragm, another for oblique illumination, a third for reflected light, etc.

Microscope. II. With a more complex microscope he makes drawings with a camera lucida, uses a lieberkuhn, and measures various distances with stage and eye-piece micrometers.

Polariscope. A simple form of polariscope with a piece of common glass for a polarizer is used with plates of selenite, crystals and compressed and unannealed glass.

Cross Hairs. This experiment consists in the insertion of cross hairs in the eye-piece of a common telescope or microscope. It is done by unravelling a common silk thread, and stretching two filaments at right angles to each other on a circle of card-board, which is then fastened to the diaphragm of the eye-piece. Considerable tension is needed to render them straight, and as they are nearly as fine as a spider's web the student acquires a good deal of delicacy of touch in handling them.

Cathetometer. This instrument is placed opposite a bent tube in which a short column of mercury balances a long column of water. Their lengths are measured, and from this the specific gravity of the mercury is deduced. A barometer tube is next filled and its height measured, deducing the level of the mercury by the usual method with a pointed steel rod. The result is finally compared with a standard barometer.

The above are some of the most successful experiments we have used, of which the whole number exceeds forty, and will probably be nearly doubled during the coming year. In selecting experiments, care is taken to introduce some which shall illustrate general methods of research, such as the micrometer screw, verniers, interpolation, computation of probable errors, etc. The graphical method is very largely used, both from its value in studying laws, and from the facility it affords for determining the accuracy of the results. As stated above, when practicable, two curves are drawn on the same sheet; one obtained by experiment, the other by computation. The student has thus the most convincing proof of the correctness of his theory.

The scale on which this laboratory has been tried, is such as to render its success no longer a matter of conjecture. In October last, the Third Year's Class, numbering twenty-three students, were admitted into it, and have since then had thirty-six exercises of one hour each. In March, the Second Year's Class, of twenty-four students, entered it, and have had twelve exercises. The total number, including special students, amounts to nearly sixty. The Third Class performed about four hundred experiments, the Second Class two hundred; the greater relative number of the latter being mainly due to the fact that the apparatus was in more perfect working order after being used for some months. Most of the experiments described above occupy but one hour, although such as the microscope and spectroscope require a longer time. Accordingly, an average student could perform all the experiments of the above series in less than thirty exercises of one hour each, and would thus acquire a far more practical knowledge of physics than by devoting the same time to lectures. Since the regular course of manipulations at the Institute is more than double this number, it is believed that each student will acquire as extended a knowledge of practical physics as is needed for everyday life.

The degree of accuracy attained in these experiments is

much greater than was anticipated, and led to the belief that investigations could be carried on, in which most of the experimental portion should be done by students. Such work would be of the utmost value to them, since, while learning to use the apparatus, they would be initiated into the methods of conducting researches, while the results being obtained by several independent observers, would be free from personal errors. To test this plan, experiments were made to determine the degree of accuracy attainable with the hook gauge, the calibration of gas meters, the perfection of compensation of the so-called "cycloid" of gas holders, and various other questions. The results will be published elsewhere, and are so encouraging that this method will be greatly extended during the coming year.

Students in the higher classes who wish to pursue Physics further, after finishing the above course, perform more difficult experiments, and carry on original investigations. As an example of the work thus accomplished, see an article by Mr. Chas. R. Cross, On the Focal Length of Microscope Objectives, in the Journal of the Franklin Institute, for June, 1870.

When a student wishes to pursue a special branch of physics, or to study a single instrument, the course is arranged to suit the requirements of each case. For example, a student, wishing to learn to use the spectroscope, would first perform the experiments of the general course bearing on this subject, such as measurement of the angles of prisms, of the power of telescopes, insertion of cross-hairs, determination of the index of refraction of different rays, and the wave lengths of the more prominent lines of the spectrum. He would also use the spectroscope, as described above, only with a greater variety of substances. After attaining some proficiency with this instrument, in which measurements are made with a photographed scale, a second spectroscope with graduated circle is given him, and with it he measures as before, the principal lines of the solar spectrum. He then constructs a curve, in which vertical distances represent scale-readings and horizontal wave-

lengths, and thus has a means of reducing all his measurements to the normal spectrum. This method is then applied to electric spectra, obtained by a Ruhmkorff coil. The lines of gases are obtained by Gessler tubes, those of metals by using them as terminals, also by drawing the spark from the surface of solutions of their salts. These measurements are reduced to wave lengths by the curve, and platted as before. In this experiment he learns to prepare a battery, and to use the induction coil. Next, he uses the large spectroscope belonging to the Institute (which produces a dispersion equal to eleven prisms of 60°), and with this he compares parts of the solar spectrum with Angstrom's chart, new lines being measured by the spider line micrometer, and interpolated from those on the map. He also reverses spectra and projects images on the slit with this instrument. The projection of the solar and metallic spectra on a screen and the spectrum-microscope will be added hereafter, and eventually the means of observing astronomical spectra, especially those of the solar protuberances. This example is selected since it has been performed substantially, as here described, by a student in spectroscopy.

Although the large number of colleges springing up all over the country has caused quite a demand for physicists, yet heretofore there has been no place where a young man could be specially educated for this profession, and consequently instructors have, in many cases, been selected from those who previously have devoted but little attention to this subject. To teach physics properly it is necessary that the instructor, besides possessing a knowledge of its phenomena and laws, which can be obtained from text-books and lectures, should be a skillful experimenter; and he should be able to construct, or at least superintend the construction of apparatus, since most institutions are unable to afford to purchase large quantities of that already made. If, as is usually the case, the instruction is given by lectures, he must be able to speak fluently, and to perform difficult experiments in the presence of an audience.

This course, therefore, in addition to the lectures and manipulations described above, will offer an opportunity to the student to perform all the more common lecture room experiments, and especially the projection of photographs and other objects on the screen, using the electric and other lights. After each lecture given to the regular students of the Institute, he will be expected to repeat all the experiments there tried. Special instruction will be given in designing apparatus and preparing estimates of its cost. It so often happens that a physicist is dependent on his own resources for most of his apparatus that students will be urged to make various instruments themselves, and will be shown how gas and steam fittings may be used, and how much may be accomplished by the aid of the carpenter and tinman.

To give him ease in addressing an audience, exercises will be held in which each student in turn will present essays on various physical subjects illustrated by experiments, care being taken to select such as will be of value to all members of the class.

The fourth object of the laboratory is to supply a place where investigations of a high order can be carried on. A physicist can here have water, gas, steam, and apparatus of all kinds, which could elsewhere be obtained only at great expense. The large size of one of our rooms (nearly an hundred feet in length) enables many experiments to be tried on a much larger scale than is usually practicable, while assistants can be obtained from among the more advanced pupils of the laboratory with mutual advantage to both parties.

In conclusion, it seems scarcely necessary to point out the advantages of the laboratory system of teaching physics, since the tendency of all technical education is now in this direction. In Chemistry, especially, this method has proved so successful as to have almost superseded instruction by lectures, at least in the larger schools and colleges.

Respectfully submitted,

EDWARD C. PICKERING,

Thayer Professor of Physics.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

REPORT OF THE SECRETARY,

INCLUDING

REPORTS OF PROF. PICKERING ON THE PHYSICAL LABORATORY, AND OF PROF. WATSON ON THE MUSEUM OF DESCRIPTIVE GEOMETRY AND MACHINERY.

In conformity with article 16, section 4, of the By-Laws of the Corporation, I herewith present the Annual Report of the transactions and condition of the Institute for the ninth year, 1870-1871.

There have been held during the year fourteen meetings of the Society of Arts; these have been well attended, and many interesting communications have been presented, as follows:

The meetings of November 3d and 17th, 1870, were taken up by the appointment of a committee to draw up a new code of By-Laws for the Society, the report of the same, and the nomination of officers.

Nov. 17. Mr. F. E. Stimpson made a communication on the economy of gas burners.

Dec. 1. Mr. R. M. Copeland made a communication on the utilization of sewage, showing what a great source of agricultural wealth is not only thrown away in large cities, but is actually converted into a producer of disease by contaminating the soil, the water, and the air.

Dec. 15. After the election of the committees constituting the Council of the Society, Mr. H. P. Langley read a paper, with illustrations, on "Cast Iron, and the manufacture of Rodman guns."

Mr. Stimpson explained a diagram of the Savery engine used for pumping out mines, introductory to a description and illustration of an automatic steam-pump for dwelling houses, attached to the ordinary range or kitchen stove.

Jan. 5, 1871. Prof. C. H. Hitchcock, of Dartmouth College, read a paper, illustrated by diagrams and a wooden model, on the

topography and geology of the Mt. Washington group in the White Mountains, and gave an account of the meteorological establishment on Mt. Washington in winter.

Jan. 19. Mr. F. E. Stimpson illustrated the efficacy of various gas burners, with photometric experiments.

Feb 2. Mr. Rowell exhibited the "Electro-pneumatic Protector," applied to a safe, which is claimed as a perfect protection against burglars. The safe is surrounded by a case whose double walls are exhausted of air; whenever this vacuum is interfered with y opening the door, or making even the finest perforation in the case, an electric current is closed, ringing an alarm; when the vacuum exists, certain spring disks are separated, and the circuit is open; the wires are so arranged that they cannot be tampered with without ringing the alarm.

Mr. Gaffield read a paper on the "U. S. Weather Reports and Storm Signals for the benefit of Commerce," describing the instruments used and the methods of observation, and showing how by this knowledge of a coming storm many lives and much valuable property can be saved.

Feb. 16. Mr. Albon H. Bailey read a paper on a practical system of logotype composition to facilitate printing, substituting syllables and short words for the usual single-letter types.

Mr. Brown explained some of the recent improvements in the type-setting machine of his invention.

March 2. Mr. C. R. Cross gave a history of spectroscopy, and of its uses in Chemistry and Astronomy; after which Prof. Pickering described the phenomena of the solar eclipse of Dec. 22, 1870, as observed in Spain by the American Expedition to which he was attached, giving an account of the experiments with the photometer by Mr. Ross, and with the polariscope—the general conclusion being that the corona is solar, and not terrestrial in its origin.

March 16. Mr. James Hamblett, Jr., gave a description of Mr. Tilghman's process for etching on glass and stone by means of a blast of sand, driven by water or steam power. It is believed that this will find numerous applications in decorative and constructive art.

Mr. Brayton exhibited in operation his compound gas engine, the power being derived from the expansion produced by the

burning of a compound of atmospheric air and coal gas, in the proportion of 10 to 1. Its advantage is that it secures continuous combustion, without explosion and consequent necessity of constant re-ignition.

April 6. Mr. H. McMurtrie read a paper, illustrated by a working machine and by diagrams, on an automatic steam Vacuum Pump.

Prof. Pickering described the Euharmonic organ, the invention of Mr. Allen, of Newburyport, in which temperament is done away with, and each musical note is given with absolute purity.

April 20. Mr. E. W. Bowditch described the various routes surveyed for an inter-oceanic canal across the Isthmus of Darien, and more particularly one to which he was attached last year, under Lieut. Selfridge, U. S. N.

Mr. J. H. Gerry explained the mechanism of keyless or stem-winding and setting watches, and presented to the Institute a set of the mechanical movements for this purpose, invented by himself, and used by the Howard Watch Co., of this city.

May 3. Prof. Richards gave an account, with apparatus, of Bunsen's air or filter pump, drawing out air by a stream of water, by which the usually slow process of filtering is much accelerated.

Mr. Gannett described the methods of determining longitudes or differences of time by the electric telegraph, showing their great accuracy.

May 18. Mr. B. H. Locke, a student of the Institute, gave the methods of investigation and the results of a long series of experiments, made in the Physical Laboratory by himself and several other students, to determine the best coverings for steam pipes, to prevent radiation, and secure the most perfect transmission of the heat or the power of steam. Three pipes were used, one uncovered, one covered with a white cement furnished by the Salamander Cement Co., and the third with hair felt. The amount of water condensed per day in one foot of uncovered pipe was about 4 1-2 lbs., in the cement covered pipe 2 1-4, and in the felt covered one 1 lb.; in other words, the coal wasted per year would be in the three cases, 13,000 lbs. for the uncovered pipe, 7,000 for the cement, and 3,000 for the felt, per hundred feet of pipe.

Prof. Pickering showed the results of some experiments made by Mr. W. T. Leman, a student of the Institute, on a new method of proving the law of falling bodies, by letting a plate of smoked glass fall in front of the prongs of a vibrating tuning fork, and then measuring the curves thus obtained.

Prof. Watson exhibited a model of the Slide Valve, and showed by diagrams the mode of determining the lap and lead at any position of the crank.

There have been elected during the year eleven associate members, the list now comprising 353 members. The re-organization of the Society of Arts, the adoption of its new code of By-Laws, and the regulation of its affairs by the Council, composed of the Committees on Communications, on Publication, on Membership, and on Finance, four committees of five each, have infused new life into its meetings; and there has been no lack of interesting and valuable material, as the above abstract shows. When the objects of the Society are better known, there can be no doubt that all persons interested in the application of science to the useful arts will recognize its value as a tribunal before which to present their nventions and discoveries, and will be glad to increase its sphere of usefulness by joining its ranks.

The Society invites all who have any valuable knowledge of this kind, which they are willing to contribute, to attend its meetings, and become members. All persons having valuable inven- ions or discoveries, which they wish to explain to an appreciative audience, will find a suitable occasion in the Society meetings, subject to proper regulations; and, while the Society will never endorse, by vote or diploma, or other official recognition, any in- ntion, discovery, theory, or machine, it will give every facility to those who wish to discuss the principles and intentions of their own machines or inventions, and will endeavor at its meetings, or through properly constituted committees, to show how far any communications made to it are likely to prove of real service to the community.

The School of Industrial Science has had a very prosperous and satisfactory year, 250 students having attended its sessions: 95 in the 1st year, 65 in the 2d, 45 in the 3d, 32 in the 4th, and the remainder special students in Chemistry, Drawing, and Architecture, including five females. Of these a little more than one-half are regular students; more than two-thirds are from Massachusetts, principally from Boston and vicinity. Thirty professors and teachers are connected with the School; the fees from students have amounted to over $31,000, nearly $6,000 more than last year, when the number of students was 224.

The apparatus for instruction has been considerably increased; a reading room and reference library have been opened under the supervision of a lady, and have proved a great success; a larger room and more books will soon be provided to make this department of the school what it should be.

Beside the ordinary courses of the school, the Trustee of the Lowell Institute has established under the supervision of the Institute of Technology, courses of instruction, generally in the evening, open to students of either sex, free of charge. This year were given

A course of eighteen lessons on Elementary French, by Prof. Bôcher.

A course of eighteen lectures on Physiology and Hygiene, by Prof. Kneeland.

A course of eighteen lectures on Modern History, by Prof. Atkinson.

A course of fifteen lectures on Descriptive Geometry, by Prof. Watson.

A course of fifteen laboratory exercises in Chemistry, by Profs. Richards and Nichols.

A course of fifteen laboratory exercises in Qualitative Analysis, by Profs. Nichols and Richards.

These courses, which are intended to provide substantial teaching, rather than merely popular illustration of the subjects, have been well attended by persons coming with a serious purpose of improvement. The average attendance has been about 100. The new hall of the Institute, capable of seating about 1,000 persons, will be finished before the opening of the next session of the

School, and will afford a very pleasant, commodious and accessible hall for the probably much larger audiences of the ensuing winter. The programme of subjects, and the extent of the courses, will be made known in October.

In addition to the regular courses in Civil, Mechanical and Mining Engineering, Chemistry, Architecture, and Science and Literature, provision has been made for a full study of Natural History, illustrated by the collections of the Boston Society of Natural History. Shorter courses will also be given preparatory for teaching Science, for business, special technical work and for the study of medicine, embracing Chemistry, Metallurgy, Physics, Natural History in all its branches, Physical Geography, Drawing, the Modern Languages, the use of the Microscope and of instruments of precision.

Among the departments of the school which have assumed during the year large and very important proportions, may be mentioned the Physical Laboratory. To establish a laboratory for teaching Physics experimentally, as Chemistry has long been successfully taught, had for years been the ardent desire of Prof. William B. Rogers, late President of the Institute of Technology, and before his retirement he had sketched and inaugurated a plan for such a laboratory at the Institute, the first in the country, since carried out and expanded by Prof. Edward C. Pickering, his successor in the Chair of Physics in the Institute. This has been in successful operation for two or three years, and is recognized by all progressive teachers of Physical Science as a great improvement in this department of education. That the Institute took a great and much needed step in advance, when it established its Physical Laboratory, is fully proved by the fact that similar Laboratories are springing up in various parts of the country, and probably will soon be attached to all the principal colleges of the North and West.

To show what the Institute's Physical Laboratory has done, and aims to do, Prof. Pickering's report is here introduced.

Report of Prof. Pickering on the Physical Laboratory.

In the Department of Physics the Institute was one of the first to adopt the Laboratory system, by which, in addition to attending the usual course of lectures on this science, each student performs a variety of experiments, and learns the methods, and to use the principal instruments of physical investigation. The system adopted during the last two years is this: A number of experiments are prepared, and the apparatus necessary for each is placed on a table, together with a complete written description. When the class enters the laboratory, each is assigned his place, by putting a card bearing his name opposite a second card representing the experiment. He then goes to the proper table, reads the description, and perhaps completes the experiment without any aid from the instructor, who is thus left free to go from desk to desk and see that no mistakes are made. When an experiment is completed, the pupil reports the result, the position of his card is changed, and he goes on as before. The number of experiments being always greater than that of the students, no delay is incurred. When practicable, the results are represented graphically by drawing two curves on the same piece of paper, one representing the numbers obtained by experiment, the other those computed by theory. Their agreement furnishes the most conclusive evidence of their correctness, and impresses on his mind the physical law which they express. As examples of these experiments, we may refer to the measurement of the deflection of beams under varying loads, the conjugate foci of lenses, their curvature, specific gravity, wave lengths, the method of using the microscope, spectroscope, polariscope, and meteorological instruments.

In addition to these we always endeavor to have several investigations in progress, of which the experimental portion shall be performed by the students. Among those recently tried, are the calibration of a standard tenth of a cubic foot, the comparative delicacy of different polariscopes, and the hook gauge compared with a simple point for measuring the surface of a liquid. In this way the students learn to carry on investigations, while at the same time much valuable work is done; and, being performed by a number of entirely unprejudiced observers, we get results free from

all personal bias, and in this respect preferable to any that could be obtained by a single experimenter.

Our more advanced pupils also carry on more difficult researches; during the last two months we have had a great many experiments made to determine what form of covering is best suited to protecting steam pipes from loss of heat by radiation. Similar pipes, covered with different materials, were subjected to steam of the same temperature and pressure, and the rate of cooling measured. Of course this was greatest in the uncovered pipe, and the percentage in each of the others was determined. The same experiment was also tried with short pipes filled with boiling water. Next, the amount of water condensed in each in a given time was weighed, and finally the volume received every minute, for half an hour, was measured. One of the students then collected the whole series of experiments, each of which is accompanied by a curve, and wrote a memoir explaining the whole subject, and showing what conclusions are to be drawn from them. Most of these experiments were performed by our students in Mechanical Engineering, and we intend next year to introduce for these students a regular course of experiments of this kind, including the pressure of steam at different temperatures, the strength of materials, weighing and measuring of all kinds, adjustment of instruments, and all other branches of technical physics which relate to this profession. The mistakes often committed by a beginner do but little harm in a laboratory where they are easily corrected, while in actual practice they often involve much loss, both of time and money. So much is this the case that often, if a student obtained no result, his time would be profitably spent in teaching him what errors he must thereafter avoid. As an example of a different kind of research, another student has been trying a new method of measuring the velocity of falling bodies, by means of a tuning fork, which draws a curved line on smoked glass. He devised the apparatus, which was made under his direction at the Institute, then drew the curves, measured them with a microscope, and compared them with those given by theory; and his instrument now becomes a valuable one to introduce in our regular course of physical manipulations.

One of the most important objects of the laboratory is to pre-

pare teachers of Physics. A large number of Institutions, where the value of a practical knowledge of the subject was realized, have applied to us for instructors, and we are very desirous of supplying this demand to the best of our ability. The course for such students would include the method of using ordinary lecture room apparatus; the adjustment of instruments, planning of apparatus, and, when possible, overseeing its construction; and finally the preparation of lectures, and proper methods of delivering them. They would, of course, be enabled to see how the regular lectures and laboratory exercises were conducted at the Institute, and, when they wished it, to assist in them. There is at the present time a great demand for good teachers of Physics, and the profession offers an excellent opening to any young man whose talents and inclinations lead him in this direction.

Still another object of the Laboratory, and perhaps the most important from a scientific point of view, is to afford professional physicists, and others engaged in conducting physical investigations of any kind, the means of performing their experiments at the Institute. The want of apparatus prevents almost any individual from doing much work of this kind, which can easily be performed in a laboratory properly supplied with water, gas, steam, and other appliances commonly needed. As instances of this kind of work, we may mention some experiments now in progress to determine accurately the change in elasticity of iron due to magnetism, and our expectation of an elaborate series of measurements of the strength of different forms of electro-magnets.

The Laboratory has heretofore been treated as an experiment, on which, therefore, but little money has been spent. The method, however, proving so successful, and being adopted in so many Institutions, both at home and abroad, enables us now to look at it from quite a different point of view. What is now needed is the means of procuring instruments of *precision*, by which all the higher portions of the subject may be treated, and with which students may make their measurements with all the accuracy needed in real observations. It was feared that the injury to such apparatus from rough handling would be very great, but experience has shown that we have little to apprehend from this source. We have obtained during the past year an admirable collection of

apparatus of this kind for measuring electrical resistances, suitable for instructing students in almost all the questions which may occur in connection with submarine cables.

Apart from their practical importance, these experiments have another value of quite a different kind, namely, as a means of general culture. Besides illustrating certain physical laws, they show the student how these laws are obtained, the true relation between theory and practice, and by what processes of reasoning our knowledge of physical science was acquired. But it is unnecessary to dwell further on these matters, as they appeal not merely to the professional Physicist, but to all believers in the importance of a sound knowledge of applied science.

The Department of Mechanical Engineering has also received important additions during the year, and the facilities offered to students will be understood from the following

Report on the Museum of Descriptive Geometry and Machinery, by Prof. William Watson.

The collections of this Museum consist of models in wood, in metal and in plaster, besides lithographs, photographs and manuscript drawings, chiefly selected from the best collections of France, Germany and Switzerland, and, in some instances, made expressly for the school. They may be grouped as follows : —

Plaster Models.

I. *Descriptive Geometry, and its applications to Shades, Shadows and Linear Design.*

1. A set of models in relief, illustrating the proper use of light and dark lines in linear design.

2. A set of models illustrating the theory and practice of shades and shadows.

3. A set of models showing the sections of single curved, double curved, and warped or twisted surfaces.

4. A set of models showing the intersections of cylinders, cones, and surfaces of revolution with each other, the penetrations made in each surface and the common solid.

II. *Masonry and Stone Cutting.*

1. Plate bands.
2. Full centred, segmental, conical, conoidal and annular arches.
3. Portals, right, oblique, and rampant.
4. Niches.
5. Trompes and brackets.
6. Domes.
7. Spiral, suspended, and covered staircases (*vis Saint Gil les.*

III. *Experimental Mechanics.*

1. Casts of Saint Vennant's models, showing the changes of forms which bodies of various shapes undergo, when subjected to forces causing flexure and torsion.
2. A full sized model of the liquid vein observed and measured by Poncelet and Lesbros, in their hydraulic experiments. These models are duplicates of those made for the *Conservatoire des Arts et Métiers*, at Paris.

IV. *Graphical Representation.*

1. A model representing the mean temperature of a place for the twenty-four hours of each day of the twelve months of the year.
2. Topographical models, showing contour lines, with accompanying topographical drawings.

MODELLING IN PASTEBOARD.

The instruction in Descriptive Geometry is practical as well as theoretical. When the drawings relating to the intersection of developable surfaces are completed, the students cut patterns of these surfaces, and rolling or folding them together produce the exact solids in space, with the apertures formed by the passage of a portion of one solid through the other. A considerable number of models thus made have been added to the collection.

MODELLING IN PLASTER. (DESCRIPTIVE GEOMETRY.)

The graphical solutions of problems relating to the sections and intersections of doubled curved surfaces are applied to solids in plaster, prepared expressly for this purpose, and the students are required to execute their solutions in relief.

Modelling Tools, Moulding and Modelling in Plaster. (Stereotomy.)

Sets of modelling tools have been provided and practical exercises in stone cutting are given; these consist in executing from rough pieces of plaster, models of portals, arches, domes, staircases, bridges, etc., with the aid of drawings and patterns previously prepared by the students themselves: in this way the collections have been increased to some extent, and it is thought that the practical skill and familiarity with the details of construction thus acquired, will ultimately be of signal service to the student in the subsequent practice of his profession.

Moulding in plaster is also taught to a limited extent.

Models in Wood or Metal.

I. *Descriptive Geometry.*

1. A set of models to illustrate Descriptive Geometry with Schröder's construction plates. They consist of models in relief of the various problems of Descriptive Geometry, arranged upon sets of planes at right angles to each other, and containing the corresponding graphical solutions.
2. Models executed in brass and silk threads to illustrate the course on developable and warped surfaces.

II. *Carpentry.*

1. Models of joints and mouldings.
2. Models of roof trusses in wood and iron, including a model illustrating Polonceau's system of iron roofs, centres for bridges, girders, etc.
3. Models of bridges.

III. *Mechanism.*

1. Models showing the different methods of laying out teeth of wheels in the various cases of racks, outside and inside gearing, etc. Bevel and skew bevel wheels.
2. An instrument for laying out teeth devised by Schröder.
3. Models of pulleys and wrapping connectors, belts and chains.
4. Models of parallel motions, including Watts' parallelogram,

applied to land and marine engines. Seward's parallel motion, fitted to the engines of the Gorgon, etc.

The foregoing models in mechanism were presented by Hon. E. B. Bigelow, of Boston.

5. Models of non-circular, and screw wheels.

6. Endless screws.

7. Wheels in trains; epicyclic trains; Ferguson's paradox; equation clock; system of Lahire, etc.

8. Models of cams.

9. Models of silent feed motions.

10. Models of quick return motions.

11. Regulating apparatus, i. e., apparatus for stopping, reversing or modifying the motions of machines. These include governors, friction cones and clutches, reversing gear, Oldham's coupling, etc.

IV. *Resistance of the Materials used in Construction.*

A set of models illustrating the best forms of beams for resisting flexure, torsion and compression under various conditions of stress; to which is added an apparatus for testing the deflections caused by loads applied in any manner to test their strength or stiffness. This collection, made Schröder, is the gift of Hon. E. B. Bigelow.

V. *Construction of Machines.*

These consist of a number of highly finished models made by Schröder, and presented by Hon. E. B. Bigelow. They include models of the parts of machines, such as screws, chains, hooks, riveting, axles, plumber blocks, steps and supports for shafts, wheels, pulleys, cranks, eccentrics, cross-heads, connecting rods, working beams, valves, pistons, etc.

VI. *Lifting Engines.*

Including the following working models:

1. Crab engine.

2. A complete model of Fairbairn's plate iron dock crane; presented by Hon. E. B. Bigelow.

3. Hydraulic press.

VII. *Hydraulic Motors.*

1. A model of the water pressure engine at Alt Mordgrube, in Freiberg, Saxony, with the pumps and apparatus for draining mines.
2. A model of Poncelet's water wheel.
3. A model of Fourneyron's turbine; presented by Hon. E. B. Bigelow.
4. A model of Jonval's turbine.
5. Swain's and Leffel's inward flow turbines.

VIII. *Steam Engines.*

1. Boilers and Fire grates.
2. Steam cylinders, pistons, valves, etc.
3. Slide valves and the mechanism, showing the distribution of the steam.
4. Variable cut-off valves — Stephenson link motion.
5. Models of steam engines of various forms.

Use of the Models.

The foregoing enumeration would be incomplete without some reference to the use made of the models and the methods of instruction in the Department of Mechanical Engineering.

Besides the ordinary lectures and recitations, there are, in this department, two distinct kinds of instruction; the first is that given in the drawing rooms in making sketches and finished drawings of machinery from models; the second is the practical instruction by projects. These projects, given in connection with the lectures and complementary to them, are of three kinds. The projects of the first kind comprise those in applied Cinematics, having for their object to determine from the graphical representation of the motion, the form adapted to each piece of mechanism.

Two or three examples taken from the actual work of the students are here inserted as illustrations:

Cams. Show how to construct a cam which shall give any assigned motion to a moving piece. Given $s = f(t)$.

Example. Make a cam which shall give exactly the motion of an eccentric, with a throw of ten inches.

Bevel Wheels. Two axes intersecting at an angle of 80° are to be connected by a pair of bevel wheels : one axis is required to make three revo-

lutions to one of the other. The largest radius of the pinion is 12 in.; this pinion is to have 44 cast iron teeth. The teeth of the wheel are to be of wood, and the width measured along the pitch surfaces is to be 5 in. The profiles of the teeth are to be involutes of the circle ; and finally there must always be at least one pair of teeth on the first in contact with a pair on the second wheel. Show how to lay out the teeth.

Skew Bevel Gearing. It is required to connect two perpendicular axes not situated in the same plane by a pair of skew bevel wheels. The shortest distance between the axes is 12 in.; the least diameter of the pinion must be 24 in.; it must have 72 teeth 6 in. in width, measured along the pitch surface. The profiles of the teeth are to be formed by epicycloids and hypocycloids.

Valve Gear. In a simple slide valve gear, let the eccentricity be 2.3 in., and the angle of lead 30°. Let the steam be cut off at .8 of the stroke, and let the release take place at .96. It is required to find the inside, and outside lap and lead, the greatest opening of the ports, and the opening of each port corresponding to any position of the crank.

The students are required to present full sized working drawings together with a memoir containing the description, the theory and practical details of the work.

These projects include the construction of cams, eccentrics, link work, and all kinds of gearing. Projects of the second kind are exercises in the construction of parts of machines, such as axles, cranks, valves, pistons, and finally of complete machines, from numerical data. And for this purpose, liberal use is made of the collections furnished by Mr. Bigelow, and enumerated above.

Projects of the third kind are not given until the students have been made acquainted with the doctrine of the strength of materials, so as to be able to find the dimensions of pieces to resist flexure, shearing, torsion, etc. They consist of original designs for machines, involving the determination of the strength, dimensions, and proper proportions of the several parts by calculation.

The following are some of these projects :

1. Project for a travelling crane to be employed in the construction of a stone bridge.
2. Project for a hydraulic foundery crane to raise twenty tons.
3. Project for a turbine, having given the fall and the volume of water.

4. Project for a set of boilers for a pumping engine of 300 horse power.

5. Project for a rolling mill driven by a steam engine.

These projects comprise —

1. The plans, elevations and sections of the machines.

2. The working drawings of the details.

3. A memoir containing the description and theory of the machines; the estimation of the resistances; the calculation of the strength and proper proportions of the parts, and the reasons for the particular dispositions adopted.

Much value is attached to these last exercises, and the whole o the previous work is made tributary to them.

In conclusion, attention is called to the three-fold use of these models; first, in the drawing rooms as objects from which sketches and finished drawings are made; second, in the lecture rooms, to illustrate the principles of machinery, and to exhibit to the eye what would otherwise require long and tedious explanations; and third, in the practical exercises in construction and design, which would be difficult, if not impossible, without them.

Sixteen students successfully passed the examinations for degrees, and will receive their Diplomas on presentation of acceptable theses in September.

In 1868, there were 27 applicants for admission to the School of the Institute; in 1869, 32; in 1870, 37; and this year, 58.

June 15, the President, four Professors, and fifteen of the graduates and fourth year's students in Mining Engineering and Metaluurgy, started for Colorado, to spend the vacation in examining mines and metallurgical processes, chiefly in Colorado; the iron region of Missouri and the mines of Utah will also be visited. This is not a pleasure excursion, but an expedition for systematic work; the students will make reports of their special investigations, which, with those of the Professors, will hereafter be submitted to the Corporation, giving, it is believed, important results in relation to the mineral wealth and economic processes of the regions visited.

Respectfully submitted,

SAMUEL KNEELAND, *Secretary.*

Boston, June 16, 1871.

[From the Journal of the Franklin Institute.]

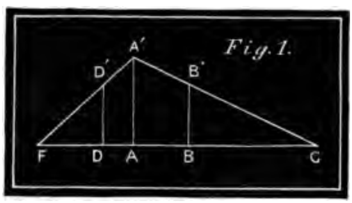
A GEOMETRICAL SOLUTION OF SOME ELECTRICAL PROBLEMS.

By Prof. Edward C. Pickering.

The analytical solution of the following problems is always a little complex, considering the simplicity of the results, and therefore unsatisfactory to the student. It is hoped that the following geometrical method may prove both useful for purposes of instruction, and suggestive of a simple solution of other more difficult problems.

The method employed is to represent by horizontal distances or abscissas, electrical resistances, and by ordinates the potentials of the various parts of an electrical circuit. Thus in fig. 1, neglecting the lines to the left of AA′, suppose AC equals the total resistance of the circuit, and that the battery has a potential AA′. If, then, one pole is connected with A, and the other with the ground at C, the potential of any point B may be found by drawing the straight line A′C, and erecting the perpendicular BB′.

Wheatstone's Bridge.—Four resistances, M, N, O and P, are connected together, end to end, two opposite junctions being connected with the poles of a battery, and the other two with a galvanometer. The needle of the latter will not be deflected when M : N=P : O. To prove this, lay off fig. 1, AB=M, BC=N, ED=O and DA=P. Suppose the battery connected at A, erect the perpendicular AA′ equal to its potential, and draw the lines AC and AE. Draw also the lines BB′ and DD′. They will represent the potentials at the terminals of the galvanometer, and will be equal if no current passes. But



AA′ : BB′=M+N : N and AA′ : DD′=P+O : O, and if BB′=DD′, M+N : N=P+O : O, hence M : N=P : O.

II. *Poggendorff's Method of Measuring Potentials.*—Let E′ equal the potential of the battery to be tested, and E the potential of that with which it is to be compared, and which is taken as a standard. The galvanometer is connected with the latter battery, interposing

a resistance to reduce the current. Now connect the two terminals of the other battery with the galvanometer, so as to pass a current through it in the opposite direction, and vary the interposed resistance until the needle comes to zero. Call G the resistance of the galvanometer, and R that of the standard battery and resistance; then there will be equilibrium when $E : E' = G + R : G$. To

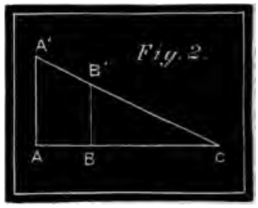


Fig. 2.

prove this, let AA′, fig. 2, equal E, $AB = R$, and $BC = G$. Let DD′ also equal E′. For equilibrium DD′ must be equal and opposite to BB′. But $AA' : BB' = AC : BC$ or $E : E' = G + R : G$.

III. *Thomson's Method of Measuring the Resistance of a Battery.*—This method, although much more recent than the others, is scarcely of less importance. The galvanometer is connected with the battery, and a resistance R interposed, to reduce the deflection. R is then removed and the galvanometer shunted, until precisely the same deflection is again produced. Call B the resistance of the battery, R′ that of the shunt, and G that of the galvanometer. Then $B = \frac{RR'}{G}$. In fig. 3, let AA′ equal the potential of the battery, $AB = B$, $BD = R$, $DE = G$. The current passing through the galvanometer will then in

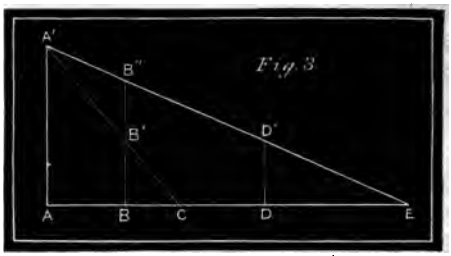


Fig. 3

the first case be measured by DD′. When the galvanometer is shunted its resistance is reduced $\frac{GR'}{G+R'}$. Lay off BC equal to this quantity. The current in the second case evidently equals BB′, and since the deflection is the same in both cases, we must have $DD' = BB'$. Now $AA' - DD' : DD' = B + R : G$ and $AA' - BB' : BB' = B : \frac{GR'}{G+R'}$. Hence $B + R : G = B : \frac{GR'}{G+R'}$, or $GB(G+R') = GR'(B+R)$, $G^2B + GBR' = GBR' + GRR'$ and $GB = RR'$. Hence $B = \frac{RR'}{G}$.

In the same way the deflection which will be produced in any given case may be determined, or the resistance which should be inserted to render it a maximum. The many applications of this method are, however, so obvions that further illustration seems unnecessary.

May 13*th*, 1873.

APPLICATIONS

OF

Fresnel's Formula

FOR THE

REFLECTION OF LIGHT.

BY

EDWARD C. PICKERING,
THAYER PROFESSOR OF PHYSICS
IN THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

EXTRACTED FROM THE PROCEEDINGS OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES.

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VOL. IX.

PAPERS READ BEFORE THE ACADEMY.

I. APPLICATIONS OF FRESNEL'S FORMULA FOR THE REFLECTION OF LIGHT. By Prof. Edward C. Pickering.

Read, Oct. 14, 1873.

Part I. Theoretical.

One of the most beautiful applications of the Undulatory Theory was made by Fresnel, in deducing a formula for computing the amount of light reflected by the surface of a transparent medium. He showed that if the light was polarized in the plane of incidence, that the amount reflected would be $A = \frac{\sin^2 (i - r)}{\sin^2 (i + r)}$, while, if polarized in a plane perpendicular to it, the proportion would be $B = \frac{\tan^2 (i - r)}{\tan^2 (i + r)}$, i and r representing the angles of incidence and refraction respectively. Natural light may be regarded as composed of two equal beams polarized at right angles, hence the amount reflected $R = \frac{1}{2} (A + B) = \frac{1}{2} \left(\frac{\sin^2 (i - r)}{\sin^2 (i + r)} + \frac{\tan^2 (i - r)}{\tan^2 (i + r)}\right)$, a formula which may be applied to any special case, by substituting proper values for i and r. The value of A evidently increases as i varies from 0° to 90°. That of B, on the other hand, diminishes from 0° until $i + r = 90°$, when it equals 0, or at this angle, which is that of total polarization, all of the ray B is transmitted, all the reflected beam being polarized in the plane of incidence. When $i = 90°$, $A = 1$, $B = 1$, hence all the light is reflected.

When the light, instead of passing from the rare to the dense medium, goes in the other direction, the same amount of light is

reflected, as we merely make $n' = \frac{1}{n}$, and i becomes r, and r, i, the values of A and B being unchanged.

When $i = 0°$, or is infinitesimal, $A = \frac{\sin^2 (n-1)r}{\sin^2 (n+1)r} = \left(\frac{n-1}{n+1}\right)^2$ and $B = \left(\frac{n-1}{n+1}\right)^2$; hence the reflected ray $R = \left(\frac{n-1}{n+1}\right)^2$ and is unpolarized. Table I. gives the values of R for various values of n.

TABLE I.

Light reflected when $i = 0°$, or incident light normal to surface.

n	$\frac{1}{2}(A+B)$	n	$\frac{1}{2}(A+B)$
1.00	0.00	2.0	11.11
1.02	0.01	2.25	14.06
1.05	0.06	2.5	18.37
1.1	0.23	2.75	22.89
1.2	0.83	3.	25.00
1.3	1.70	4.	36.00
1.4	2.78	5.	44.44
1.5	4.00	5 83	50.00
1.6	5.33	10.	66.67
1.7	6.72	100.	96.08
1.8	8.16	∞	100.00
1.9	9.63		

The light therefore increases with n, being zero when $n = 1$, and 100 per cent, or all reflected when $r = \infty$.

This law also holds for other angles of incidence, and serves to explain many familiar phenomena. The brilliancy of the diamond is mainly due to its high index, causing it to reflect about 20 per cent of the light falling normally on it, while glass returns but 4 per cent.

The white color of fine powders is generally due to the light reflected from the faces of the minute crystals of which they are composed. Hence, immersing them in a liquid diminishes their brightness, since the relative index being reduced, less light is reflected. This is seen when snow is immersed in water, or when we write with wet chalk on a blackboard. In the last case, the relative index $= \frac{\text{index chalk}}{\text{index water}} = \frac{1.5}{1.33} = 1.167$, and 0.6 per cent only is reflected; while, when dry, the index is 1.5, and 4 per cent, or six times as much, is sent back. For the same reason, white lead is injured as an oil paint when adulterated with sulphate of baryta, which, although as white, has an index of refraction of 1.68 instead of 1.9. The effect is to diminish the covering power, the light being transmitted instead of reflected, so that when applied as a paint more coats are needed. In the four cases, we have white lead,

dry, index = 1.9, reflects 9.6 per cent; white lead in oil, index 1.27, reflection 1.44; baryta, dry, index 1,68, reflection 6.9; baryta in oil, index 1.12, reflection 0.32. Accordingly, when dry, either reflects enough light, while, when surrounded with oil, the baryta is nearly transparent. Baryta, or even carbonate of lime, may, however, be used as a water color, since either has a large enough index compared with air. With colored paints, on the other hand, we wish to destroy the reflected light which, united with the natural color of the pigment, deadens it. Hence oil colors are more brilliant than water, and the latter brighter when wet than dry. Numerous other facts may be explained in the same way, as that varnishing increases the brilliancy of pebbles and wood; in diamond mines the rough gem is distinguished from the quartz pebbles, which it resembles, by immersing it in water; and paper is rendered transparent by oiling it.

Let us next discuss the case where n is very nearly unity, or $n = 1 + dn$. Since $\sin i = n \sin r$, $\cos i \, di = \sin r \, dn$, and $di = \frac{\sin r}{\cos i} dn = \text{tang } i \, dn$. Now $i - r = di$, and $i + r = 2i + di = 2i$ $\therefore$ $A = \left(\frac{\text{tang } i \, dn}{\sin 2i}\right)^2 = \left(\frac{dn}{2\cos^2 i}\right)^2 = \frac{dn^2}{4}\sec^4 i = \frac{dn^2}{4}(1 + \text{tang}^2 i)^2$, and $B = \left(\frac{\text{tang } i \, dn}{\text{tang } 2i}\right)^2 = \left(\frac{dn}{2}(1 - \text{tang}^2 i)\right)^2 = \frac{dn^2}{4}(1 - \text{tang}^2 i)^2$, and the degree of polarization of the reflected beam, or $\frac{A - B}{A + B} = \frac{2\,\text{tang}^2 i}{1 + \text{tang}^4 i} = \frac{2 \sin^2 i}{2 - \sin 2 i}$.

Of course the absolute amount of light reflected when dn is infinitesimal is zero, unless $i = 90°$; but as commonly we only wish to compare the relative amounts when dn is small, it is generally best to neglect the constant term $\frac{di^2}{4}$, and use $A = (1 + \text{tang}^2 i)^2 = \sec^4 i$, and $B = (1 - \text{tang}^2 i)^2 = \left(\frac{\text{tang } i}{\text{tang } 2i}\right)^2$. When di varies, the reflection is proportional to its square, or to dn^2. When $i = 0$, A, B, and R become equal to 1; hence this forms an excellent unit, with which the amount reflected at other angles of incidence may be compared.

Table II. gives the amount of light for various angles of incidence. Column 1 gives i, the angle of incidence; column 2 gives A; column 3, B; column 4, $\frac{1}{2}(A + B)$, or the amount of light reflected; and column 5, the degree of polarization.

This table is of interest, as it is applicable to all cases where the two surfaces have nearly the same index. It will be noticed that the angle of total polarization is 45°, and that as i approaches 90°, the amount of light reflected becomes very great.

TABLE II.

Light reflected when n is near unity, or equals $1 + dn$.

i	A	B	$\frac{1}{2}(A+B)$	$\frac{A-B}{A+B}$
0°	1.000	1.000	1.000	0.0
5	1.015	.985	1.000	1.5
10	1.063	.939	1.001	6.2
15	1.149	.862	1.005	14.3
20	1.282	.752	1.017	26.0
25	1.482	.612	1.047	41.5
30	1.778	.444	1.111	60.0
35	2.221	.260	1.240	79.1
40	2.904	.088	1.496	94.5
45	4.000	.000	2.000	100.0
50	5.857	.176	3.016	94.5
55	9.239	1.081	5.160	79.1
60	16.000	4.000	10.000	60.0
65	31.346	12.952	22.149	41.5
70	73.079	42.884	57.981	26.0
75	222.85	167.16	195.00	14.3
80	1099.85	971.21	1035.53	6.2
85	17330.64	16808.08	17069.36	1.5
90	∞	∞	∞	0.0

The most important application of Fresnel's formula is to the case of glass, where n somewhat exceeds 1.5. Brewster calculated such a table for $n = 1.525$, which is published in Phil. Trans. 1830, p. 143. A much more complete table is given below (Tables III., IV., and V.)

TABLE III.

Light reflected when $n = 1.55$.

i	r	A	B	dA	dB	$\frac{1}{2}(A+B)$	$\frac{A-B}{A+B}$
0°	0° 0'.0	4.65	4.65	.130	.130	4.65	0.0
5	3° 13'.4	4.70	4.61	.131	.129	4.65	1.0
10	6° 25'.9	4.84	4.47	.135	.126	4.66	4.0
15	9° 36'.7	5.09	4.24	.141	.121	4.66	9.1
20	12° 44'.8	5.45	3.92	.150	.114	4.68	16.4
25	15° 49'.3	5.95	3.50	.161	.105	4.73	25.9
30	18° 49'.1	6.64	3.00	.175	.094	4.82	37.8
35	21° 43'.1	7.55	2.40	.191	.081	4.98	51.7
40	24° 30'.0	8.77	1.75	.210	.066	5.26	66.7
45	27° 8'.5	10.38	1.08	.233	.049	5.73	81.2
50	29° 37'.1	12.54	.46	.263	.027	6.50	92.9
55	31° 54'.2	15.48	.05	.303	.007	7.74	99.3
60	33° 58'.1	19.35	.12	.342	—.013	9.73	98.8
65	35° 47'.0	24.69	1.18	.375	—.032	12.91	91.2
70	37° 19'.1	31.99	4.00	.400	—.050	18.00	77.7
75	38° 32'.9	42.00	10.38	.410	—.060	26.19	61.8
80	39° 26'.8	55.74	23.34	.370	—.069	39.54	41.0
85	39° 59'.6	74.52	49.03	.250	—.061	61.77	20.6
90	40° 10'.7	100.00	100.00	.000	—.000	100.00	0.0

Angle of total polarization = 57° 10'.3, A = 16.99.

TABLE IV.

Light reflected when $n = 1.55$, and i is near angle of total polarization.

i	B	dB
50°	.463	+.027
51	.359	+.023
52	.263	+.019
53	.179	+.015
54	.119	+.011
55	.053	+.007
56	.016	+.003
57	.000	—.001
58	.008	—.005
59	.037	—.009
60	.120	—.013

TABLE V.

Light reflected when $n = 1.55$, and i is near 90°.

i	A	B	dA	dB	i	A	B	dA	dB
70° 0′	31.99	4.00	.400	.050	86° 0′	79.02	56.62	.209	.055
71 0	33.21	4.94	.405	.052	86 10	79.80	57.98	.201	.053
72 0	35.24	6.03	.409	.054	86 20	80.58	59.39	.193	.052
73 0	37.38	7.28	.411	.056	86 30	81.39	60.82	.187	.050
74 0	39.64	8.72	.412	.058	86 40	82.18	62.29	.178	.049
75 0	42.00	10.38	.410	.060	86 50	82.98	63.79	.171	.047
76 0	44.49	12.31	.406	.062	87 0	83.80	65.32	.163	.046
77 0	47.08	14.54	.400	.064	87 10	84.63	66.89	.155	.045
78 0	49.80	17.07	.392	.066	87 20	85.46	68.49	.147	.043
79 0	52.66	20.00	.383	.068	87 30	86.31	70.14	.140	.041
80 0	55.74	23.34	.370	.069	87 40	87.15	71.82	.133	.040
80 30	57.36	25.20	.361	.069	87 50	88.01	73.55	.125	.038
81 0	59.04	27.18	.353	.069	88 0	88.88	75.31	.118	.036
81 30	60.77	29.33	.342	.068	88 10	89.76	77.11	.109	.034
82 0	62.60	31.57	.331	.068	88 20	90.64	78.96	.099	.032
82 30	64.41	34.04	.320	.067	88 30	91.54	80.75	.090	.030
83 0	66.30	36.64	.310	.067	88 40	92.44	82.79	.081	.027
83 30	68.27	39.43	.297	.066	88 50	93.35	84.76	.072	.025
84 0	70.29	42.41	.284	.065	89 0	94.28	86.79	.063	.022
84 30	72.37	45.61	.267	.063	89 10	95.21	88.88	.054	.019
85 0	74.52	49.03	.250	.061	89 20	96.16	91.00	.045	.015
85 10	75.25	50.23	.244	.060	89 30	97.11	93.18	.035	.012
85 20	75.99	51.45	.238	.059	89 40	98.06	95.40	.024	.008
85 30	76.74	52.69	.232	.058	89 50	99.02	97.66	.012	.004
85 40	77.49	53.96	.224	.057	90 0	100.00	100.00	.000	.000
85 50	78.25	55.28	.217	.056					

for $n = 1.55$, the letters having the same meaning as in Table II. Columns 5 and 6 furnish a means of determining A and B for any other value of n. They represent the change in these quantities, for a

change of n of .01. For instance, the value of A for $n = 1.55$, $i =$ 25°, is 5.95, as given in the table, and it increases .161 for every increase of .01 in n. Thus, when $n = 1.50$, $A = 5.15$. In the same way B then equals 2.98. These values are of course only approximate, and are most correct for values of n between 1.50 and 1.55. In the same way, Table IV. gives the corresponding values of B, and $d\,B$ for angles near that of total polarization, and Table V. A and B for values near 90°.

In practice we commonly have to deal with some even number of parallel surfaces, especially several plates of glass. It is therefore important to discuss this case for various values of i, and of the number of plates. When a surface reflects the fraction A of the light, the transmitted portion equals $1 - A$; and if there is no internal reflection, the second surface will reflect the same proportion, or $A\,(1 - A)$, and transmit $(1 - A)^2$. In the same way, m surfaces will transmit $(1 - A)^m$. But practically, of the light reflected by the second surface, part will be turned back by the first, so that the total transmitted ray equals $\frac{1 - A}{1 + A}$, and that reflected $\frac{2\,A}{1 + A}$. In the same way it may readily be proved that for m surfaces the transmitted ray equals $\frac{1 - A}{1 + (m - 1)\,A}$, and the reflected ray $\frac{m\,A}{1 + (m - 1)\,A}$. Table VI. gives the values of i, A, and B, the amount of light reflected, the amount polarized by reflection, and that polarized by refraction, for 1, 2, 8, and 20 surfaces. The amount refracted of course equals 100 minus that reflected. The absorbed light is here neglected, as it is comparatively small, and varies with each specimen of glass.

The following conclusions are readily drawn from an examination of the numbers in Table VI., or, better, from the curves given in Figs. 1 and 2. In all the figures accompanying this paper, ordinates represent percentages, and abscissas angles of incidence. In Fig. 1, the four highest curves represent the polarization of the beams reflected by 1, 2, 8, and 20 surfaces. The other four curves give the corresponding refracted beams. Fig. 2 gives all the curves of Table VI., relating to twenty surfaces; the five curves corresponding to A, B, the intensity of the refracted beam, and the polarization of both the reflected and refracted beams. When $i = 0$, both the reflected and refracted beams are unpolarized. With ten plates of glass about half the light is reflected, the transmitted ray being but little brighter than that reflected. With 1 or 2 surfaces the reflected beam increases as i increases; with 8 surfaces it remains nearly constant up to 50°; while with 20 surfaces a marked diminution is perceived. This very remark-

TABLE VI. — Light reflected by 1, 2, 8, and 20 Surfaces.

	1 Surface.					2 Surfaces, 1 Plate.				
i	*A*	*B*	Reflect.	Per ct. Polariz.		*A*	*B*	Reflect.	Per ct. Polariz.	
				Reflct.	Refrct.				Reflct.	Refrct.
0°	4.6	4.6	4.6	0.0	0.0	8.8	8.8	8.8	0.0	0.0
5	4.7	4.6	4.6	1.0	0.0	9.0	8.7	8.8	1.0	0.1
10	4.8	4.5	4.7	4.0	0.2	9.2	8.6	8.9	4.1	0.4
15	5.1	4.2	4.7	9.1	0.4	9.8	8.1	8.9	9.2	0.9
20	5.4	3.9	4.7	16.4	0.8	10.4	7.5	8.9	16.3	1.6
25	5.9	3.5	4.7	25.9	1.3	11.1	6.7	8.9	25.4	2.5
30	6.6	3.0	4.8	37.8	1.9	12.5	5.8	9.1	36.4	3.6
35	7.5	2.4	5.0	51.7	2.7	14.2	4.7	9.4	49.3	5.2
40	8.8	1.7	5.3	66.7	3.7	16.4	3.5	10.0	64.0	7.2
45	10.4	1.1	5.7	81.2	4.9	19.0	2.1	10.6	80.1	9.4
50	12.5	0.5	6.5	92.9	6.5	22.4	0.5	11.4	95.6	12.5
55	15.4	0.1	7.7	99.8	8.3	26.9	0.1	14.0	99.8	15.6
57	17.0	0.0	8.5	100.0	9.3	29.1	0.0	14.5	100.0	17.0
60	19.3	0.1	9.7	98.8	10.6	32.4	0.2	16.3	98.8	19.2
65	24.7	1.1	12.9	91.2	13.5	38.4	2.5	20.4	87.7	23.2
70	32.0	4.0	18.0	77.7	17.1	48.5	7.7	28.1	72.6	28.4
75	42.0	10.4	26.2	61.8	21.9	59.6	19.7	39.1	52.2	33.2
80	55.7	23.3	39.5	41.0	26.8	71.7	38.0	54.8	30.7	37.2
82	62.6	31.6	47.1	32.8	29.3	77.1	48.3	62.7	22.7	38.6
84	70.3	42.4	56.3	24.7	32.1	82.7	60.2	71.4	15.7	39.6
85	74.5	49.0	61.8	20.6	33.3	85.4	68.5	75.8	12.6	40.0
86	79.0	56.6	67.2	16.5	34.8	88.2	72.5	80.3	9.8	40.4
87	83.8	65.3	74.6	12.4	36.3	91.4	79.2	85.3	7.0	40.8
88	88.9	75.3	82.1	8.2	37.9	94.1	86.1	90.1	4.4	41.0
89	94.3	86.8	90.5	4.1	39.6	96.9	92.9	94.9	2.0	41.1
90	100.0	100.0	100.0	0.0	41.2	100.0	100.0	100.0	0.0	41.2

	8 Surfaces, 4 Plates.					20 Surfaces, 10 Plates.				
0°	28.3	28.3	28.3	0.0	0.0	49.4	49.4	49.4	0.0	0.0
5	28.6	28.0	28.3	0.9	0.3	49.7	49.1	49.4	0.6	0.6
10	29.2	27.4	28.3	3.4	1.2	50.4	48.3	49.3	2.1	2.1
15	30.2	26.3	28.3	7.4	2.7	51.5	46.9	49.2	4.7	4.6
20	31.9	24.5	28.2	13.1	5.1	53.3	44.9	49.1	8.5	8.3
25	33.9	22.4	28.2	20.3	7.9	55.8	42.2	49.0	13.9	13.4
30	36.3	19.8	28.1	29.2	11.4	58.7	38.2	48.4	21.2	19.7
35	39.5	16.5	28.0	41.1	16.0	62.2	33.0	47.6	30.9	27.9
40	43.7	12.1	27.9	56.4	21.8	65.9	26.3	46.2	42.4	36.7
45	48.3	7.4	27.9	73.3	28.1	69.8	17.1	43.4	55.9	46.6
50	53.5	3.2	28.3	88.7	35.1	74.1	8.5	41.3	79.5	56.4
55	59.4	0.5	30.0	98.3	42.0	78.4	1.0	39.7	97.5	64.5
57	62.1	0.0	31.0	100.0	45.0	80.4	0.0	40.2	100.0	67.2
60	66.3	1.7	34.0	95.0	48.9	82.8	2.4	43.6	94.5	71.0
65	72.5	7.6	40.0	81.0	54.1	86.9	18.6	52.7	64.7	72.2
70	79.0	25.0	52.0	51.9	56.3	90.4	45.5	68.0	33.0	70.0
75	85.3	48.2	66.7	27.8	55.8	93.5	69.9	81.7	11.3	64.6
80	91.0	71.1	81.0	12.3	52.6	96.2	85.9	91.0	5.6	57.1
82	93.1	79.0	86.0	8.2	50.5	97.1	90.3	93.7	3.6	54.0
84	95.0	85.5	90.2	5.2	48.6	97.9	93.7	95.8	2.2	50.9
85	95.5	88.4	92.0	3.8	47.5	98.3	95.1	96.7	1.6	49.3
86	96.8	91.3	94.0	2.6	46.2	98.7	96.4	97.5	1.2	47.7
87	97.6	93.8	95.7	1.7	45.0	99.0	97.4	98.2	0.8	46.1
88	98.4	96.1	97.2	1.1	43.8	99.4	98.4	98.9	0.5	44.5
89	99.2	98.1	98.6	0.5	42.5	99.7	99.2	99.4	0.2	42.9
90	100.0	100.0	100.0	0.0	41.2	100.0	100.0	100.0	0.0	41.2

able result may be expressed by saying that 10 plates of glass transmit more light obliquely than normally. The appearance to the eye confirms this result, but it deserves a careful photometric proof. At 57° the reflected ray is of course, in all cases, totally polarized; but at other angles the amount of polarization is greater the less the number of surfaces, instead of the contrary, as might have been anticipated.

With the refracted ray quite a different law holds. For 1 surface the polarization increases from 0° to 90°; with 2 surfaces it becomes sensibly constant near 90°; while with a larger number a distinct maximum is obtained. It is commonly supposed that the greatest effect is obtained at the angle of total polarization. But the maximum is sensibly beyond this, unless a very large number of plates is employed; and hence it seems probable that a bundle of plates, polarizing by refraction, would give the best results if set at a greater angle than 57°, as 65° or 70°. The transmitted ray, however, diminishes rapidly for large angles of incidence. A very large number of plates is required to render the polarization nearly complete, which accounts for the light always remaining when even the best polariscopes by refraction are crossed. At 90° all the refracted beams are polarized by the same amount of 41.2 per cent. Or, at grazing incidence, the amount of polarization is independent of the number of plates, one polarizing as completely as a hundred. This number 41.2 may be obtained as follows. Differentiate the value of A in terms of i and r, and make $i =$ 90°, when the refracted beam will equal $1 - A = 4 \tan r \, di = 3.376$ di, since when $i = 90°$, $r = 40° \, 10'.7$. In the same way $1 - B = \frac{8}{\sin 2r} di = 8.115 \, di$, and applying to them the formulæ for the polarization of the refracted beam, we find it equal to 41.2.

The last portion of Table VI. was determined in part by graphical interpolation, and hence the results are less accurate than those of the other tables. It is believed, however, that the errors are much less than those of any known method of testing them experimentally, and hence give all the accuracy needed for practical purposes.

To show how far these effects are due to the internal reflection, Table VII. has been computed for the same number of surfaces, supposing that no secondary reflection takes place. The first column gives the angle of incidence, the next three the polarization of the reflected, and the last three that of the refracted beam. A comparison with Table VI. shows that while the reflected beam is affected but little, a great change takes place in the transmitted light. A simple method of expressing the difference between Tables VI. and VII. is to

say that, in the former, the transmitted rays are computed by the formula $\frac{1-a}{1+(m-1)a}$, and in the latter, where the internal reflection is neglected, by the formula $(1-a)^m$.

TABLE VII.

Polarization, supposing there is no internal reflection.

	Reflected Beam.			Refracted Beam.		
i	2.	8.	20.	2.	8.	20.
0°	0.0	0.0	0.0	0.0	0.0	0.0
30	87.0	82.8	24.1	3.8	15.2	36.5
45	80.3	75.1	64.0	9.9	37.6	75.6
57	100.0	100.0	100.0	18.4	68 2	95.2
70	74.5	54.8	28.4	33.2	88.1	99.8
80	32.2	6 3	0.0	50.0	97.6	100.0
90	0.0	0.0	0.0	70.1	99.8	100.0

Part II. Experimental.

To test the above conclusions, two experimental methods may be employed. First, by means of a photometer, to determine the amount of light in any given case; and, secondly, by means of a polarimeter, to determine the percentage of polarization of the reflected and refracted rays. The latter method has been employed in the following experiments. The instrument commonly used to measure the amount of polarization was invented by Arago, and is called a polarimeter. It consists of a Nicol's prism and Savart's plates, in front of which are several glass plates, free to turn, and carrying an index which moves over a graduated circle, thus showing the angle through which they have been rotated. The prism and plates form a Savart's polariscope, which gives colored bands with either light or dark centre, according as the plane of the prism is parallel or perpendicular to the plane of polarization. When the plates are so placed that the light passes through them normally, they have no effect on it; but when turned, they polarize it in a plane parallel to the axis of rotation, and by an amount dependent on the angle. Let the instrument be so set that the axis of rotation shall be perpendicular to the plane of polarization, and the plates set at zero. The bands will then be visible, the centre one being bright. As the plates are turned, the bands become fainter, until their polarization neutralizes that originally present in the beam; beyond this point the bands reappear dark-centred. The amount of polariza-

tion is thus readily determined, by turning the plates until the bands disappear, when the angle is reduced to percentages by means of a table. The difficulty of computing this table is, however, the real objection to the use of this instrument. It may be determined by the formulæ given in the first part of this paper, but it of course then fails to prove them. Moreover, no account is taken of imperfect transparency, dust on the surface, and other sources of error. An excellent way of forming this table experimentally is to view through the instrument a beam of light totally polarized. If now the plane of polarization of the beam is changed, the percentage of polarization will alter, being zero when it is inclined 45° to the axis of the plates, and wholly polarized at an angle of 0° or 90°. At any angle a, the beam may be regarded as composed of two, $\cos^2 a$ polarized vertically, and $\sin^2 a$ polarized horizontally. The percentage of polarization will therefore equal $\frac{\cos^2 a - \sin^2 a}{\cos^2 a + \sin^2 a} = \cos 2\,a$, from which the polarization corresponding to any given angle is readily determined.

The result of such a comparison is given in Table VIII. Four series of observations were taken, of which the numbers obtained are given in Table XIII. From them curves were constructed, with angles of incidence as abscissas and percentages of polarization as ordinates. A curve was next drawn, coinciding with them as nearly as possible, and its ordinates are given in Table VIII., column 3; the angles of incidence are given in the first column, and the theoretical polarization in the second column of the same table. Column 4 gives the differences, and from it we see that, while the agreement is very close between 0° and 60°, above this point a marked variation is perceptible. This deviation will be further discussed in connection with Table XIII. and Fig. 8.

TABLE VIII.

Table for Arago's Polarimeter.

i	Theoretical.	Empirical.	Difference.
0°	0.0	0.0	0.0
20	5.1	5.0	—0.1
30	11.4	13.0	1.6
40	21.8	23.5	1.7
50	35.1	37.0	1.9
55	42.0	43.0	1.0
60	48.9	49.0	0.1
65	54.1	57.5	3.4
70	56.3	63.5	7.2
75	55.8	67.0	11.2
80	52.6	72.0	19.4

To avoid the defects of the above instrument, the following arrangement has been employed. A brass tube, *A B*, Fig. 3, about a foot long, is closed at one end by a double image prism, *B;* and at the other by a rectangular aperture, *A*, of such a width that its two images, as in the Arago polariscope, shall be in contact, but not overlapping. To the prism is attached a Nicol's prism, free to turn, and carrying an index, moving over a graduated circle, which shows how far it has been rotated. The tube is then mounted, so that it can be set at any altitude or azimuth, or rotated around its own axis, and three graduated circles serve to measure these quantities. In the instrument as actually constructed (Fig. 4), the whole is supported on an upright, which terminates below in a large screw, *C*, by which it may be attached to a post or tree, when used out of doors. A tube slips over this, which carries a cross-piece forming a **T**, and through the top of this passes the end of a second **T**, through the end of which the polarimeter slides. Three of these tubes are graduated to show the azimuth and altitude of the polarimeter tube, and the amount it is turned around its own axis.

The working of the instrument is as follows. If the Nicol's prism is removed, and the light unpolarized, the two images of the aperture at the end will be equally brilliant. If now the Nicol's prism is replaced and turned, the images will vary in brightness, alternately disappearing at intervals of 90°. If the light is polarized, one image will in general be brighter than the other; but by turning the Nicol's prism, certain positions will always be found in which the two images will be equal. The percentage of polarization is then readily determined from the angle through which the prism has been turned. To determine the law which connects these two, let the plane of polarization be vertical, and the line of junction of the two images parallel to it. Then call A and B the brightness of the two images respectively, in which case the polarization $p = \frac{A-B}{A+B}$. If the prism is turned through an angle v, one image will have a brightness $A \sin^2 v$, the other $B \cos^2 v$; and if they are equal, $A \sin^2 v = B \cos^2 v$, hence $p = \frac{\cos^2 v - \sin^2 v}{\cos^2 v + \sin^2 v} = \cos 2v$. The amount of polarization is then very simply found by turning the Nicol until the images are equal, then reading the angle, doubling it, and taking the cosine. Evidently there are four positions of equality of the image; and in the following experiments all four were observed, reading to tenths of a degree, and the mean taken. To make the reduction, Table IX. is employed, in which the columns headed p give the percentage of polarization corresponding to the angle a.

TABLE IX.

Table for Polarimeter: $p = \cos 2a \times 100$.

a	p	a	p	a	p	a	p
0.0°	100.00	5.0°	98.48	10.0°	93.97	15.0°	86.60
0.1	100.00	5.1	98.42	10.1	93.85	15.1	86.43
0.2	100.00	5.2	98.36	10.2	93.73	15.2	86.25
0.3	99.99	5.3	98.29	10.3	93.61	15.3	86.07
0.4	99.99	5.4	98.23	10.4	93.48	15.4	85.90
0.5	99.98	5.5	98.16	10.5	93.36	15.5	85.72
0.6	99.98	5.6	98.10	10.6	93.23	15.6	85.54
0.7	99.97	5.7	98.03	10.7	93.11	15.7	85.35
0.8	99.96	5.8	97.96	10.8	92.98	15.8	85.17
0.9	99.95	5.9	97.89	10.9	92.85	15.9	84.99
1.0	99.94	6.0	97.81	11.0	92.72	16.0	84.80
1.1	99.93	6.1	97.74	11.1	92.59	16.1	84.62
1.2	99.91	6.2	97.67	11.2	92.45	16.2	84.43
1.3	99.90	6.3	97.59	11.3	92.32	16.3	84.24
1.4	99.88	6.4	97.51	11.4	92.19	16.4	84.06
1.5	99.86	6.5	97.44	11.5	92.05	16.5	83.87
1.6	99.84	6.6	97.36	11.6	91.91	16.6	83.68
1.7	99.82	6.7	97.28	11.7	91.77	16.7	83.48
1.8	99.80	6.8	97.20	11.8	91.64	16.8	83.29
1.9	99.78	6.9	97.11	11.9	91.50	16.9	83.10
2.0	99.76	7.0	97.03	12.0	91.35	17.0	82.90
2.1	99.73	7.1	96.94	12.1	91.21	17.1	82.71
2.2	99.70	7.2	96.86	12.2	91.07	17.2	82.51
2.3	99.68	7.3	96.77	12.3	90.92	17.3	82.31
2.4	99.65	7.4	96.68	12.4	90.78	17.4	82.11
2.5	99.62	7.5	96.59	12.5	90.63	17.5	81.91
2.6	99.59	7.6	96.50	12.6	90.48	17.6	81.71
2.7	99.56	7.7	96.41	12.7	90.33	17.7	81.51
2.8	99.53	7.8	96.32	12.8	90.18	17.8	81.31
2.9	99.49	7.9	96.23	12.9	90.03	17.9	81.11
3.0	99.45	8.0	96.13	13.0	89.88	18.0	80.90
3.1	99.42	8.1	96.03	13.1	89.73	18.1	80.70
3.2	99.38	8.2	95.93	13.2	89.57	18.2	80.49
3.3	99.34	8.3	95.83	13.3	89.41	18.3	80.28
3.4	99.30	8.4	95.73	13.4	89.26	18.4	80.07
3.5	99.25	8.5	95.63	13.5	89.10	18.5	79.86
3.6	99.22	8.6	95.53	13.6	88.94	18.6	79.65
3.7	99.17	8.7	95.42	13.7	88.78	18.7	79.44
3.8	99.12	8.8	95.32	13.8	88.62	18.8	79.23
3.9	99.07	8.9	95.21	13.9	88.46	18.9	79.02
4.0	99.03	9.0	95.11	14.0	88.29	19.0	78.80
4.1	98.98	9.1	95.00	14.1	88.13	19.1	78.59
4.2	98.93	9.2	94.89	14.2	87.96	19.2	78.37
4.3	98.88	9.3	94.78	14.3	87.80	19.3	78.15
4.4	98.82	9.4	94.66	14.4	87.63	19.4	77.93
4.5	98.77	9.5	94.55	14.5	87.46	19.5	77.71
4.6	98.72	9.6	94.44	14.6	87.29	19.6	77.49
4.7	98.66	9.7	94.32	14.7	87.12	19.7	77.27
4.8	98.60	9.8	94.21	14.8	86.95	19.8	77.05
4.9	98.54	9.9	94.09	14.9	86.78	19.9	76.83

TABLE IX., — *continued.*

Table for Polarimeter: $p = \cos 2a \times 100$.

a	p	a	p	a	p	a	p
20.0°	76.60	25.0°	64.28	30.0°	50.00	35.0°	34.20
20.1	76.38	25.1	64.01	30.1	49.70	35.1	33.87
20.2	76.15	25.2	63.74	30.2	49.39	35.2	33.54
20.3	75.93	25.3	63.47	30.3	49.09	35.3	33.22
20.4	75.70	25.4	63.20	30.4	48.79	35.4	32.89
20.5	75.47	25.5	62.93	30.5	48.48	35.5	32.56
20.6	75.24	25.6	62.66	30.6	48.17	35.6	32.23
20.7	75.01	25.7	62.39	30.7	47.87	35.7	31.90
20.8	74.78	25.8	62.11	30.8	47.56	35.8	31.56
20.9	74.55	25.9	61.84	30.9	47.25	35.9	31.23
21.0	74.31	26.0	61.57	31.0	46.95	36.0	30.90
21.1	74.08	26.1	61.29	31.1	46.64	36.1	30.57
21.2	73.85	26.2	61.01	31.2	46.33	36.2	30.24
21.3	73.61	26.3	60.74	31.3	46.02	36.3	29.90
21.4	73.37	26.4	60.46	31.4	45.71	36.4	29.57
21.5	73.13	26.5	60.18	31.5	45.40	36.5	29.24
21.6	72.90	26.6	59.90	31.6	45.09	36.6	28.90
21.7	72.66	26.7	59.62	31.7	44.78	36.7	28.57
21.8	72.42	26.8	59.34	31.8	44.46	36.8	28.23
21.9	72.18	26.9	59.06	31.9	44.15	36.9	27.90
22.0	71.93	27.0	58.78	32.0	43.84	37.0	27.56
22.1	71.69	27.1	58.50	32.1	43.52	37.1	27.23
22.2	71.45	27.2	58.21	32.2	43.21	37.2	26.89
22.3	71.20	27.3	57.93	32.3	42.89	37.3	26.56
22.4	70.96	27.4	57.63	32.4	42.58	37.4	26.22
22.5	70.71	27.5	57.36	32.5	42.26	37.5	25.88
22.6	70.46	27.6	57.07	32.6	41.94	37.6	25.54
22.7	70.21	27.7	56.78	32.7	41.63	37.7	25.21
22.8	70.97	27.8	56.50	32.8	41.31	37.8	24.87
22.9	69.72	27.9	56.21	32.9	40.99	37.9	24.53
23.0	69.47	28.0	55.92	33.0	40.67	38.0	24.16
23.1	69.21	28.1	55.63	33.1	40.35	38.1	23.85
23.2	68.96	28.2	55.34	33.2	40.03	38.2	23.51
23.3	68.71	28.3	55.05	33.3	39.71	38.3	23.17
23.4	68.45	28.4	54.76	33.4	39.39	38.4	22.83
23.5	68.20	28.5	54.46	33.5	39.07	38.5	22.49
23.6	67.94	28.6	54.17	33.6	38.75	38.6	22.15
23.7	67.69	28.7	53.88	33.7	38.43	38.7	21.81
23.8	67.43	28.8	53.58	33.8	38.11	38.8	21.47
23.9	67.17	28.9	53.29	33.9	37.78	38.9	21.13
24.0	66.91	29.0	52.99	34.0	37.46	39.0	20.79
24.1	66.65	29.1	52.70	34.1	37.14	39.1	20.45
24.2	66.39	29.2	52.40	34.2	36.81	39.2	20.11
24.3	66.13	29.3	52.10	34.3	36.49	39.3	19.77
24.4	65.87	29.4	51.80	34.4	36.16	39.4	19.42
24.5	65.61	29.5	51.50	34.5	35.84	39.5	19.08
24.6	65.34	29.6	51.20	34.6	35.51	39.6	18.74
24.7	65.08	29.7	50.90	34.7	35.18	39.7	18.39
24.8	64.81	29.8	50.60	34.8	34.86	39.8	18.05
24.9	64.55	29.9	50.30	34.9	34.53	39.9	17.71

TABLE IX., — *continued.*

Table for Polarimeter: $p = \cos 2a \times 100$.

a	p	a	p	a	p	a	p
40.0°	17.36	41.3°	12.88	42.6°	8.37	43.8°	4.19
40.1	17.02	41.4	12.53	42.7	8.02	43.9	3.84
40.2	16.68	41.5	12.19	42.8	7.67	44.0	3.49
40.3	16.33	41.6	11.84	42.9	7.32	44.1	3.14
40.4	15.99	41.7	11.49	43.0	6.98	44.2	2.79
40.5	15.64	41.8	11.15	43.1	6.63	44.3	2.44
40.6	15.30	41.9	10.80	43.2	6.28	44.4	2.09
40.7	14.95	42.0	10.45	43.3	5.93	44.5	1.74
40.8	14.61	42.1	10.11	43.4	5.58	44.6	1.40
40.9	14.26	42.2	9.76	43.5	5.23	44.7	1.05
41.0	13.92	42.3	9.41	43.6	4.88	44.8	0.70
41.1	13.57	42.4	9.06	43.7	4.54	44.9	0.35
41.2	13.23	42.5	8.72				

Evidently when the light is unpolarized, the angle will be 45°; when totally polarized, 0°. We must now determine the effect when the line of junction is not parallel to the plane of polarization, but inclined at an angle w. The two images will in this case have a brightness $(A \cos^2 w + B \sin^2 w)$, and $(A \sin^2 w + B \cos^2 w)$. Hence the apparent polarization $p' = \frac{(A \cos^2 w + B \sin^2 w) - (A \sin^2 w + B \cos^2 w)}{(A \cos^2 w + B \sin^2 w) + (A \sin^2 w + B \cos^2 w)}$ $= \frac{A - B}{A + B} \cos 2w = p \cos 2w$. Hence, if the line of junction is not parallel to the plane of polarization, the observations must be reduced by dividing by $\cos 2w$. Evidently if $w = 45°$, the light appears to be unpolarized. The above discussion suggests a means of determining the direction of the plane of polarization. Make two observations of the amount of polarization, turning the polarimeter 45°. Then call p, p', p'', the true and the observed polarization in the two cases, and w the unknown angle between the line of junction in its first position and the plane of polarization. Having given p' and p'', we wish to determine p and w. Evidently $p' = p \cos 2w$, and $p'' = p \cos 2 (45° - w) = p \sin 2w$. Taking their quotient gives $\text{tang } 2w = \frac{p''}{p'}$, and the sum of their squares gives $p = \sqrt{p'^2 + p''^2}$. This method, though elegant theoretically, does not appear very accurate practically, as the plane is more accurately determined by covering the end of the polarimeter with a cap containing a plate of selenite, thus converting it into an Arago's polariscope. Then turn the tube until the two images have precisely the same color, when their line of junction will be inclined

45° to the plane of polarization. The plane may also be determined, more easily but less accurately, by removing the Nicol's prism, and turning the tube until the two images are equally bright, and adding 45° to the reading.

We next wish to determine the delicacy of this instrument in different parts of its scale. If the Nicol's prism is set at an angle v', differing slightly from its true value v, the brightness of the two images will be $A \sin^2 v'$ and $B \cos^2 v'$ respectively. Now it is commonly assumed that the difference in two such images will be perceptible, when the difference in brightness, divided by the brightness of either, equals a certain fraction $\frac{1}{a}$, in which a equals about 80. Now $\frac{1}{a} =$ $\frac{A \sin^2 v' - B \cos^2 v'}{A \sin^2 v'} = \frac{\sin^2 v' \cos^2 v - \cos^2 v' \sin^2 v}{\sin^2 v' \cos^2 v} = \frac{4 \sin (v + v') \sin (v' - v)}{\sin^2 2 v}$; hence, since v is substantially equal to v', $\frac{1}{a} = \frac{4 \sin (v' - v)}{\sin 2 v} = \frac{4\, dv}{\sin 2 v}$; again, since $p = \cos 2 v$, $dp = -2 \sin 2 v\, dv$, and $dp = \frac{1 - p^2}{2 a} = \frac{\sin^2 2 v}{2 a}$, from which the error in the result for any unobserved difference in brightness of the two images is readily determined.

If $p = 0$, $dp = \frac{1}{2 a}$, its greatest value, which diminishes as the polarization increases, becoming zero when $p = 0$. Hence the greater the polarization, the more accurately it can be measured. If $a = 80$, $dp = \frac{1}{160}$ for its greatest value; hence the instrument should always give results within two-thirds of 1 per cent. Observation, however, shows that the error is much greater, a difference in brightness of $\frac{1}{80}$ being by no means perceptible. To determine this point, ten observations of a bright unpolarized cloud were taken, and gave a probable error of 1°.1, which corresponds to 3.8 per cent. The beam reflected from a plate of glass was then observed in the same way, and the probable error was found to be 0°.7, equal 1.0 per cent. As the polarization in this case was 87 per cent, the probable error should have been 3.8 $(1 - p^2) = 3\,8 \times 2.43 = 0.9$, a result agreeing very closely with the observed amount. As might be expected, the error varies also with the intensity of the light, so that a sheet of unpolarized paper gave a probable error of 5.2 per cent. To compare the absolute brightness in this case with that of the cloud, the polarimeter was directed towards the latter, and its aperture half covered by the paper. The prism was then turned until the dark image of the sky just equalled the bright image of the paper. The mean angle was then found to be 25°, whence the relative brightness of the two images was found to be

$\text{tang}^2\ 25° = .21$. As each of the following observations is the mean of four, the probable error is reduced one-half.

The first series of observations were made on the light of the sky. The instrument was screwed into a post and levelled, the altitude and azimuth of the sun taken, and the instrument then directed towards the points to be observed. Most of these were in the same vertical plane with the sun, so that it was only necessary to determine their altitudes. The line of junction was then brought parallel to the plane of polarization; that is, turned until it was vertical, since it then lay in the plane passing through the sun. The four positions of the Nicol's prism, in which the two images were equally bright, were then observed, reading the angles to tenths of a degree, and taking the mean. The percentage of polarization was finally obtained from Table IX. In Table X., series 1 to 9 were taken at Waterville, N. H., in a valley at a height of about 1500 feet, surrounded by mountains about 4000 feet high. The air there may be regarded as very pure. Series 10 and 11 were taken upon the top of the building of the Institute of Technology, Boston. The first column in each case gives the altitude of the point observed, the second its distance from the sun, and the third the polarization corresponding to the mean of the four observations. It soon became evident that the polarization depended on the solar distance of the point under observation, and not on the altitude. This is more evident from Fig. 5, in which abscissas give the solar distances and ordinates the polarization of the above points.

TABLE X.

Sky Polarization.

Altitude.	Sun's Dist.	Polarization.
1. July 7. 6.45 A.M.		
—15	142	19.6
—30	126	38.9
—45	110	65.4
—60	95	64.9
2. July 7. 9.30 A.M.		
—20	112	66.2
—35	95	79.9
—50	80	68.1
—65	64	47.8
70	18	2.8
40	—18	2.1
25	—29	11.2
15	—40	19.3
90	85	16.9
—57	60	44.9
—47	70	64.5
—37	80	72.4
—38	88	76.9
—19	97	77.7
— 5	110	62.2
3. July 8. 7.05 A.M.		
—10	145	8.0
—20	135	16.4
—30	125	37.8
—40	115	50.9
4. July 10. 6 30 A.M.		
—15	149	18.4
—20	144	14.2
—30	133	28.6
—40	122	41.8
—50	111	55.0
—60	100	69.2
—70	90	73.8

TABLE X., — *continued.*

Sky Polarization.

5. July 15. 6.40 P.M.

Altitude.	Sun's Dist.	Polarization.
30	22	4.2
40	32	13.2
50	42	20.5
60	53	31.9
70	63	46.0
80	73	65.0
90	84	69.2
−80	94	70.9
−70	104	62.7
−60	115	53.0
−50	125	33.6
−40	135	20.1
−30	146	8.5
−20	156	2.8
−12	165	−3.5

6. July 16. 6.30 P.M.

20	12	1.4
30	22	1.7
40	33	10.8
50	44	16.7

7. July 20. 12.20 P.M.

60	− 8	7.0
50	−18	−1.0
40	−27	5.6
30	−36	11.5

8. July 30. 4.00 P.M.

15	−21	−1.0
20	−15	−2.4
25	− 9	−2.8
40	7	2.4
45	12	−2.4
50	17	2.1
60	28	6.6
−60	88	72.7
−50	98	69.7
−40	108	58.2
−30	119	48.2
−20	129	29.2
−10	139	21.5

9. July 30. 6.30 P.M.

Altitude.	Sun's Dist.	Polarization.
13	3	−2.8
18	8	−1.7
28	19	−0.3
38	29	2.1
48	40	11.8
58	50	25.9
−60	113	55.6
−50	123	41.3
−40	134	21.5
−30	144	14.6
−20	155	2.4
−10	165	−1.7
−20	156	−9.8
−30	146	−2.8
−40	137	6.3

10. Sept. 12. 9.15 A.M.

5	−40	12.2
15	−30	9.8
25	−20	5.2
66	20	3.1
76	30	10.8
86	40	10.8
−83	50	22.8
−73	60	33.9
−63	70	48.8
−52	80	57.1
−47	85	59.3
−42	90	62.7
−36	95	62.9
−31	100	57.7
−26	105	56.5
−20	110	55.0
−15	115	49.4
−10	120	43.5
− 5	125	37.1
− 0	130	31.2

11. Sept. 12. 6.00 P.M.

− 8	170	2.8
−19	160	7.7
−29	150	12.2
−40	140	23.8
−50	130	37.1
−61	120	51.5
89	90	79.0
58	60	47.2
48	50	29.9
37	40	19.1
27	30	10.1
16	20	5.6
6	10	1.7

Before discussing these observations further, it seemed desirable to determine the polarization of other parts of the atmosphere not lying in the same vertical plane with the sun. Moreover, as the polarization of points at equal distances from the sun should be compared, the polarimeter was so mounted that its principal axis would pass through the

sun. The two graduated circles would then give solar distances, and the angle from the vertical plane through the sun, instead of altitudes and azimuths. The second of these angles will be called the meridian distance, and will be regarded as positive to the right, and negative to the left of the sun. Of course, the direction of the axis should continually change, so as to follow the sun; but as great accuracy in the determination of the angles was not needed, it was found sufficient to readjust it every few minutes. Another advantage of this arrangement was, that the line of junction, being turned parallel to the axis, would always lie in the plane passing through the sun, and hence be parallel to the plane of polarization. Table XI. gives the result of five series of observations made in this way: —

TABLE XI.

Sky Polarization. Points equidistant from Sun.

M. D.	Polar.
12. July 15.	
Sun's Dist. 90°, Alt. 5°.	
75	78.6
60	74.3
45	79.8
30	77.0
0	77.2
30	74.8
45	75.2
60	76.8
75	76.8
Mean	76.7
13. July 20.	
Sun's Dist. 60°, Alt. 20°.	
0	44.1
30	41.8
60	40.7
—90	42.9
—60	45.7
—30	38.5
Mean	42.9
14. July 30.	
Sun's Dist. 90°, Alt. 48°.	
0	61.8
—30	69.0
—75	69.7
75	60.5
60	60.2
0	61.6
—55	68.7
—60	71.7
Mean	65.4
Sun's Dist. 60°, Alt. 45°.	
0	45.7
30	42.6
60	41.3
90	39.4
110	36.2
—30	49.1
—60	38.7
—90	39.7
—110	41.3
0	42.9
Mean	41.7
Sun's Dist. 120°, Alt. 42°.	
0	45.7
30	39.1
Mean	42.4
Sun's Dist. 30°, Alt. 40°.	
0	13.6
60	5.2
90	7.3
120	3.5
180	5.9
—120	19.1
—90	15.3
—60	16.0
Mean	10.7
15. Sept. 12.	
Sun's Dist. 90°, Alt. 40°.	
0	66.9
30	67.9
60	66.4
75	61.6
0	66.4
30	68.2
60	65.1
75	65.6
90	54.8
0	65.6
Mean	60.4
Sun's Dist. 120°, Alt. 41°.	
0	42.9
30	44.8
45	44.5
55	41.6
0	40.5
—30	41.8
—45	37.5
—55	35.8
0	38 1
Mean	40.9
Sun's Dist. 60°, Alt. 43°.	
0	30.9
30	35.5
60	37.5
90	33.9
120	33.2
—30	35.8
—60	31.9
—90	32.6
—120	38 4
0	31.9
Mean	34.2
16. Sept. 12.	
Sun's Dist. 90°, Alt. —5°.	
0	77.9
—30	79.4
—60	79.0
—75	77.8
0	75.0
30	75.7
Mean	77.5

The first column gives in each case the meridian distance, the second the mean of the four positions of equality of the Nicol's prism, and the third the corresponding polarization. All these observations point to one very remarkable result; namely, that the polarization is the same for a given solar distance for any meridian distance; in other words, that the polarization is the same for all points equally distant from the sun. The variations in the observations are to be ascribed partly to errors of observation and partly to real irregularities in the atmosphere, as it is evident that they follow no regular law. The means therefore give us the true polarization with much greater accuracy. They are represented in Fig. 5 by small crosses. The next thing is to determine the law which connects the polarization with the solar distance in all these observations. A drawing was made like Fig. 5 enlarged, and a fine copper wire laid on it, and bent into such a shape that it should coincide as nearly as possible with all the observations. The ordinates for every 10° were then read off, giving the results entered in column 2 of Table XII.

TABLE XII.

Theoretical Formula for Sky Polarization.

S. D.	Obs.	Theor.	Differ.
0	0.0	0.0	0.0
10	1.0	1.0	0.0
20	3.5	4.2	+ 0.7
30	9.0	10.0	+ 1.0
40	17.5	18.2	+ 0.7
50	28.5	29.0	+ 0.5
60	41.0	42.0	+ 1.0
70	56.0	55.4	— 0.6
80	67.0	66.1	— 0.9
90	72.0	70.0	— 2.0
100	68.0	66.1	— 1.9
110	58.0	55.4	— 2.6
120	44.0	42.0	— 2.0
130	30.0	29.0	— 1.0
140	18.0	18.2	+ 0.2
150	8.5	10.0	+ 1.5
160	3.0	4.2	+ 1.2
170	1.0	1.0	0.0
180	0.0	0.0	0.0

A simple explanation of the polarization of the sky is to assume that it consists of molecules of air or aqueous vapor, which reflect the light specularly, and whose index of refraction differs only by a very minute amount from that of the medium in which they float. The theoretical

polarization would then be at once given by Table II., making i equal to one-half the solar distance. The curve thus obtained is given in Fig. 5, at A. The polarization according to this should be complete at 90° from the sun, while in reality it is only about 70 per cent. If, however, we multiply the ordinates of curve A by this fraction, we obtain curve B, which agrees almost precisely with the curve given in column 2 of Table XII. Its ordinates are given in column 3, and the differences in column 4. From the latter it will be seen that the empirical curve gives results somewhat too great for solar distances less than 60°, and too small for greater distances; but the deviation is so small, compared with the accidental errors, that we are scarcely justified in drawing any conclusions from them. The agreement of all the observations in the neighborhood of 120° from the sun is remarkable, and not easily explained. The observations of series 10 for distances less than 110° give results decidedly below that given by theory. A possible explanation is the reflection of the sun on the sea to the east of Boston, a source of error not present in the earlier observations which were made inland. It will be noticed that no account is here taken of the points of no polarization, or neutral points of the sky; but the polarization is very slight for some distance from them, and hence is not easily measured. They must be regarded as due to some secondary disturbing cause, as refracted light, which alters the general polarization of the sky but little.

When the polarimeter is directed towards a polished colored plane surface, the two images assume different tints. One, which contains the light polarized in the plane of incidence, or B, is composed mainly of the light reflected specularly, and is therefore white like the source of light. The image A contains but little of the light reflected specularly, consisting principally of the rays emitted by the body, and hence partaking of its color. The idea at once suggested itself, that testing the light of the sky in this way might give a clue to the cause of its color. The experiment was tried several times, with negative results, the two images appearing of precisely the same blue tint. But on the evening of July 15th, near sunset, when measuring the polarization of a point near the northern horizon, where the blue color was comparatively pale, a marked difference in the two images was observable. The image B was found to be of a yellowish brown, A of a grayish blue or violet tint. This observation has since been frequently repeated, and can, in fact, be made almost any clear evening near sunset. Evidently we may conclude from these colors that the true color of the sky particles is blue, a view quite in accordance with

the observations of Prof. Cooke with the spectroscope, and of Prof. Tyndall on aqueous vapor in a state of formation.

Observations were next made to test the results found above, for the light reflected and transmitted by several parallel surfaces of glass. To check the results, which are given in Table XIII., two, and in some cases three, independent methods were employed. For convenience of reference, the series are numbered, as in the observations on sky polarization. The general method employed to measure the polarization of the reflected ray was to lay one or more sheets of glass on a piece of black velvet, and render them horizontal with a spirit-level. The polarimeter was then mounted a short distance from them, carefully levelled and then turned down, so that the light should be reflected from their surfaces. Its angle of depression would then equal the complement of the angle of incidence. The line of junction of the two images was then rendered vertical, and the polarization measured in the usual way. The polarization of the sky, if clear, would introduce a large error into the results, and care was therefore taken to make these observations only on cloudy days. Although it is commonly stated that no traces of polarization can be detected in the heavens when completely covered with clouds, yet it was found to be slightly polarized in a vertical plane at such times, the effect being most marked near the horizon, and probably due to reflection from terrestrial objects. To obtain a single surface of glass, a piece was blackened on one side in the flame of a candle, in the expectation that the oil in the lamp-black, having nearly the same index of refraction as the surface to which it adhered, would prevent all specular reflection from it. But the results obtained did not agree with those computed for a single surface, and a close examination showed that double reflections were given of objects in front, as with a common plate of glass; in fact, that the lamp-black acted merely like the velvet in the other cases. The two series, Nos. 20 *a* and 21 *a*, are therefore placed with the observations on two surfaces, with which they agree very well. A piece of colored glass was next used, which gave the results in the column headed 17 *a*. Series 22 *a*, 27 *a*, 35 *a*, and 36 *a* were obtained in the same manner, using 1, 4, and 10 pieces of plate glass, laid on one another so as to form a pile.

The other measurements of the polarization of the reflected beam were obtained by quite a different method. A large Babinet's goniometer, or optical circle, was employed, the slit being removed and replaced by a Nicol's prism, which was free to turn around its axis, the angle of rotation being measured by a graduated circle and index. In the eye-piece of the observing telescope, a Nicol's prism was placed, and

TABLE XIII.

Observed Polarization of 1, 2, 8, and 20 Surfaces of Glass.

ONE SURFACE.

Reflected Beam.

i	17 *a*.	18 *b*.	19 *b*.	Theor.
0°	—	—	—	0.0
10	—	—	—	4.1
20	17.0	17.4	17.0	16.4
30	38.6	38.7	39.5	37.8
40	72.7	68.4	69.2	66.7
45	79.9	83.5	85.0	81.2
50	93.6	94.5	94.8	92.9
55	97.2	99.9	99.9	99.3
60	97.6	97.7	97.8	98.8
65	87.8	88.3	88.0	91.2
70	77.5	73.8	71.7	77.7
75	58.5	58.5	52.8	61.8
80	41.0	38.7	34.9	41.0
82	—	29.6	27.2	32.8
84	—	23.5	19.9	24.7
85	20.1	18.4	18.9	20.6
86	—	15.6	13.7	16.5
87	—	12.9	9.2	12.4
88	—	5.6	4.9	8.2
89	—	—	—	4.1

TWO SURFACES, ONE PLATE.

	Reflected Beam.					Refracted Beam.			
i	20 *a*.	21 *a*.	22 *a*.	23 *b*.	Th.	24 *a*.	25 *b*.	26 *b*.	Th.
0°	—	—	2.8	0.0	0.0	0.0	−0.7	1.0	0.0
10	2.5	4.5	—	—	4.1	—	2.4	1.4	0.4
20	19.7	22.1	17.0	14.9	16.3	1.4	0.0	—	1.6
30	36.0	42.3	34.9	37.8	36.4	4.9	4.9	—	3.6
40	62.5	62.9	62.9	65.6	64.0	7.3	7.7	—	7.2
45	75.1	79.4	80.3	79.9	80.1	—	9.8	—	9.4
50	87.9	88.1	92.4	93.4	95.6	11.8	11.5	—	12.5
55	95.7	94.4	98.0	99.6	99.3	14.3	14.6	—	15.6
60	93.6	93.2	97.6	98.0	98.8	15.6	19.1	17.4	19.2
65	87.1	85.9	89.3	87.5	87.7	20.1	23.2	23.8	23.2
70	69.2	71.9	71.0	71.2	72.6	26.2	27.2	26.2	28.4
75	53.9	52.4	56.2	51.5	52.2	34.2	30.9	32.2	33.2
80	30.9	37.1	33.2	35.5	30.7	34.9	36.8	39.7	37.2
82	—	—	—	22.8	22.7	31.9	39.7	41.3	38.6
84	—	—	—	20.8	15.7	—	40.3	45.7	39.6
85	16.0	19.8	18.4	21.1	12.6	39.4	41.3	48.5	40.0
86	—	—	—	—	9.8	—	46.9	50.3	40.4
87	—	—	—	—	7.0	—	49.3	56.2	40.8
88	—	—	—	—	4.4	—	49.3	56.8	41.1
89	—	—	—	4.2	2.0	—	55.3	51.2	41.2

TABLE XIII., — *continued.*

Observed Polarization of 1, 2, 8, and 20 Surfaces of Glass.

EIGHT SURFACES, FOUR PLATES.

	Reflected Beam.			Refracted Beam.						
i	27 *a.*	28 *b.*	Th.	29 *a.*	30 *b.*	31 *c.*	32 *c.*	33 *c.*	36 *c.*	Th.
0°	—	0.7	0.0	0.7	—0.3	0.0	1.7	0.5	1.2	0.0
10	4.9	—	3.4	1.4	0.3	—	—	—	—	1.2
20	16.0	14.3	13.1	6.6	5.2	4.2	9.8	—	—	5.1
30	31.9	30.6	29.2	12.9	12.5	12.9	—	—	—	11.4
40	62.9	56.5	56.4	19.8	23.5	23.8	23.5	—	—	21.8
45	75.7	73.4	73.3	27.9	31.2	—	—	—	—	28.1
50	88.1	88.3	88.7	33.5	37.5	37.1	36.5	—	—	35.1
55	96.8	99.0	98.8	41.3	43.5	45.7	—	—	—	42.0
60	92.8	96.5	95.0	50.0	51.8	49.7	48.4	52.4	53.0	48.9
65	75.5	74.8	81.0	55.6	58.8	58.5	—	57.5	59.9	54.1
70	52.1	44.8	51.9	61.6	66.4	64.3	64.0	63.5	62.4	56.3
75	33.2	20.8	27.8	67.7	71.4	64.0	66.9	65.6	66.1	55.8
80	19.8	8.0	12.3	64.3	74.5	62.9	77.5	72.7	70.5	52.6
82	—	—	8.2	66.4	81.5	—	—	79.0	75.9	50.5
84	—	—	5.2	64.5	91.3	—	—	—	—	48.6
85	11.8	5.9	3.8	61.6	89.7	—	—	—	—	47.5
86	—	—	2.6	54.2	—	—	—	—	—	46.2

TWENTY SURFACES, TEN PLATES.

	Reflected Beam.			Refracted Beam.				
i	35 *a.*	36 *a.*	37 *b.*	Th.	38 *a.*	39 *b.*	40 *b.*	41 *c.*	Th.
0°	—	—	1.4	0.0	1.7	0.2	0.0	0.7	0.0
10	—3.8	5.0	—	2.1	5.6	0.7	2.8	—2.1	2.1
20	8.7	16.3	8.0	8.5	10.8	10.1	13.6	9.8	8.3
30	29.2	25.2	16.0	21.2	25.2	20.8	28.6	22.1	19.7
40	52.4	55.0	36.5	42.4	39.4	45.1	46.3	44.8	36.7
45	70.2	68.7	51.8	55.9	53.3	56.9	62.7	54.0	46.6
50	85.8	83.3	73.1	79.5	63.7	71.9	74.3	63.7	56.4
55	95.3	96.2	97.4	97.5	70.5	82.9	85.3	76.6	64.5
60	90.6	88.5	86.1	94.5	78.4	94.3	91.6	82.1	71.0
65	62.7	65.3	67.5	64.7	81.3	97.7	93.8	85.9	72.2
70	41.8	37.5	31.9	33.0	81.1	—	97.0	91.2	70.0
75	22.5	33.9	13.9	11.3	76.6	99.1	—	94.7	64.6
80	19.8	17.0	11.3	5.6	71.9	79.6	—	85.2	57.1
85	18.0	8.6	9.4	1.6	58.2	—	—	—	49.3

in front of it quartz wedges giving lines, which were bright or dark centred, according as the transmitted ray was polarized vertically or horizontally. On looking through the telescope, the field was seen to be traversed with lines, which disappeared only when the Nicol in the collimator was inclined 45° with the vertical. At any other angle, a,

the vertical and horizontal components, were $\cos^2 a$ and $\sin^2 a$, and hence were equivalent to a beam polarized vertically by an amount $\cos 2a$. If now any object was inserted between the two telescopes polarizing the light horizontally p, the bands would disappear only when $p = \cos 2a$. Measuring the four positions of disappearance, and taking their mean, gave an accurate measure of the polarization by Table IX., using it as with the polarimeter described above. Another way of expressing the effect of this instrument is to say that the bands disappear when the Nicol is so turned that the plane of polarization shall be brought by the object under examination to an angle of 45°. The method of measuring the polarization of the reflected ray is now obvious. The pieces of glass are placed vertically on the centre plate between the two telescopes, the latter set at an angle of $2i$, and the glass turned until the light is reflected from its surface, so as to render the field bright. The Nicol is then turned until the bands disappear, and its position recorded. The angle between the telescopes is then altered so as to make i successively 20°, 30°, 40°, &c., and the observation repeated. Various adjustments must be made to eliminate constant errors, but they need not be detailed here. Series 18 *b* consists of observations thus made on a glass prism having an index of refraction of 1.517; series 19 *b* was made with colored glass; series 23 *b* was made with one sheet of plate-glass; and series 28 *b* and 37 *b* with 4 and 10 microscope slides respectively. The latter were used, as the thickness of the plate-glass was such that, when a number of plates were placed between the telescopes, a portion of the internal reflection would be lost.

To measure the polarization by refraction, two similar methods were employed. The plates were placed vertically over the centre of a graduated circle, and a piece of ground-glass was viewed through them by the polarimeter. The plates were then set at various angles, and the polarization measured in each case. All these observations were made in cloudy weather, to eliminate the effect of sky polarization. Series 24 *a*, 29 *a*, and 38 *a* were obtained in this manner. Other observations were made with the optical circle, placing the telescopes opposite each other, and recording the angles of the Nicol for various positions of the plates. The results are shown in columns 25 *b*, 26 *b*, 30 *b*, 39 *b*, and 40 *b*. Still a third method was employed, already described in connection with the Arago's polarimeter. Four series — columns 31 *c*, 32 *c*, 33 *c*, and 34 *c* — were thus measured with four slips of glass for microscopic slides, and one series, 41 *c*, with ten pieces of plate-glass.

To show more clearly which method of measurement was employed

in each series, the letter *a* is attached to all the columns measured with the polarimeter, *b* to those measured on the optical circle with Babinet's wedges, and *c* to those in which the point of disappearance of Savart's bands was found.

It will be noticed that no observations are given of the polarization of a beam transmitted by one surface of glass. There seemed to be no easy method of measuring this quantity. It might be done by making a series of prisms of such angles that when the light was incident on one face at 10°, 20°, 30°, &c., the refracted ray would strike normally on the second face. The effect of the latter would then be nothing, so that the polarization would in this case be entirely due to the first surface.

We proceed now to discuss the results given in Table XIII. Examining the observations on a single surface of glass, we see that their concordance is much greater than in the observations of sky polarization, although two quite distinct methods were employed. The column headed "Theor." gives the theoretical polarization as given in Table VI. The observed polarization is somewhat too great for angles less than 57°, and too small for greater angles. The difference may be explained by the fact that the index of refraction was somewhat less than 1.55, hence the angle of total polarization less than 57°. The same results are shown in Fig. 6, in which abscissas represent angles of incidence, and ordinates polarization. In the case of the light reflected by two surfaces, Fig. 7, the agreement is also very close. The same may be said for the refracted beam for angles below 80°. It is difficult to observe the polarization at greater angles, as the light then passes so obliquely through the glass. There seems, however, to be a decided excess of the observed over the theoretical polarization. Two series only, of observations on the light reflected by four plates of glass, Fig. 8, were taken, as they seem to agree sufficiently well with each other, and with theory. The case of the light transmitted by four plates of glass has special importance from its application to the polarimeter of Arago. For although, of course, any other number of plates might be used, yet this number, since the eclipse of 1871, seems to have been more frequently employed. Six concordant series are given, obtained by three distinct methods, and all agree in showing a marked divergence from theory for angles greater than 60°, the observed being greater than the computed polarization.

The observations on twenty surfaces, as might be expected, present still greater discrepancies. As regards the reflected ray, Fig. 9, series 37 *b* agrees pretty well with theory, but gives a much less result than

either 35 *a* or 36 *a*. Both of the latter were taken by the same method, and agree pretty well together, but differ from theory at 45° by 15 per cent. The variations of the refracted beam, Fig. 10, are still greater. As before, the observations taken by the same method as 39 *b* and 40 *b* agree, but 41 *c*, taken with the Savart, gives smaller results; series 38 *a*, with the polarimeter, still less; and theory, least of all. The errors most likely to occur, which would be common to all the observations on the refracted beams, are, first, stray light, or light entering the instrument without passing through the glass; secondly, light passing through the glass endwise, which might be recognized by its deep green color; and, thirdly, light reflected from the front surfaces of the plates. But all these errors would tend to diminish, instead of increase, the polarization; and hence, if eliminated, the divergence from theory would be still greater. Probably the true explanation is that internal reflection does not take place as completely as theory assumes, partly owing to the imperfect transparency of the medium, and partly to the dust and other impurities on the surface. Comparing the results with Table VII., which shows the effect when there is no internal reflection, we see that it makes but little difference for the reflected rays, the polarization being the same for three values of i, namely, 0°, 57°, and 90°. For the refracted ray, on the other hand, the variations are very great, amounting in the case of twenty surfaces, at 90° incidence, to over 50 per cent. We also see from Tables VI. and VII. that a partial absence of the internal reflection would account for all the results obtained, while neglecting it entirely, would cause a still greater divergence between theory and observation.

On account of the thickness of the bundle of ten plates of glass, a portion of the secondary reflection would be thrown a considerable distance to one side, especially when i is large, so that it might fall quite outside of the instrument, or even be cut off by the ends of the plates. This effect would be least marked with the polarimeter, next with the Savart, and most of all with the optical circle, on account of the small aperture of the telescope. But this is just the order in which the observations stand, all of them falling between the two theoretical curves. These observations with Tables VI. and VII. also show the effect to be expected from a bundle of plates when used to polarize light by refraction. If ten plates are employed, set, as is usual, at 57°, the polarization would be only 67.2 per cent if internal reflection takes place, but would be 95.2 if this is in any way excluded. We may, in passing, point out that an advantage might be expected in such a polariscope from an increase in the angle of incidence, the increased

polarization probably more than making up for the loss of light and distortion induced by the increased obliquity of the incident rays.

The want of perfect transparency of the glass would also tend to increase the polarization by enfeebling the secondary reflection, and dirt or grease on the surface of the glass would produce the same effect. With eight surfaces, these disturbing causes are much less marked, except for large angles of incidence, and hence the agreement with theory much better. Beyond 60°, however, it becomes perceptible, producing the increased polarization noticed above. Even with a single plate of glass, this disturbing cause becomes perceptible, which probably accounts for the divergence for angles greater than 80°.

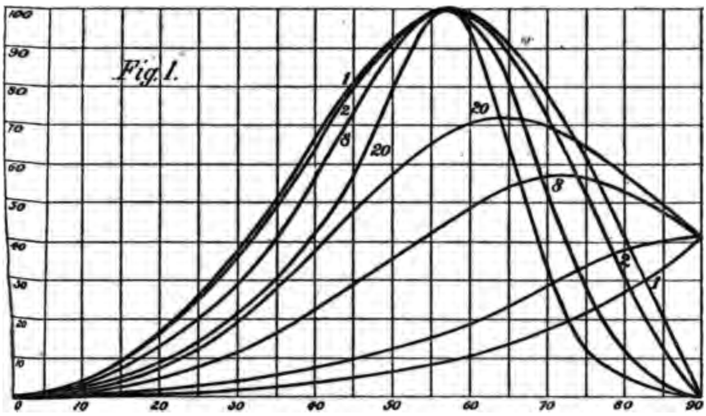


Fig. 1.

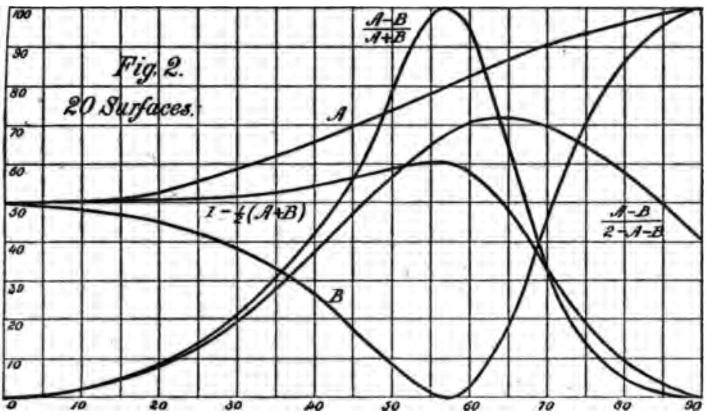


Fig. 2.

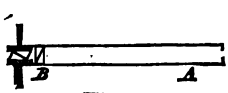


Fig. 3.

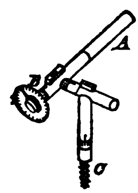


Fig. 4.

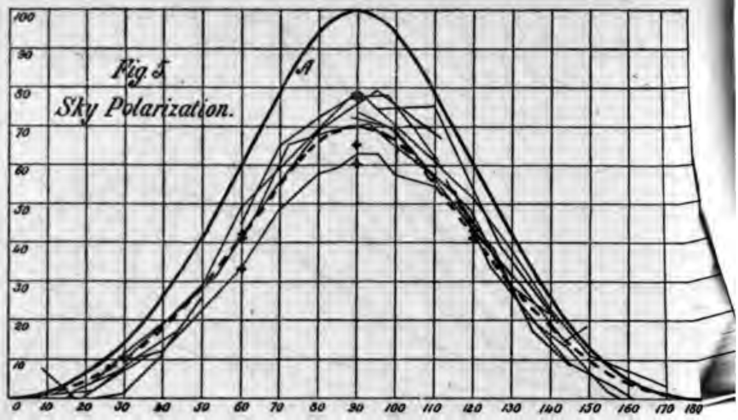


Fig. 5. Sky Polarization.

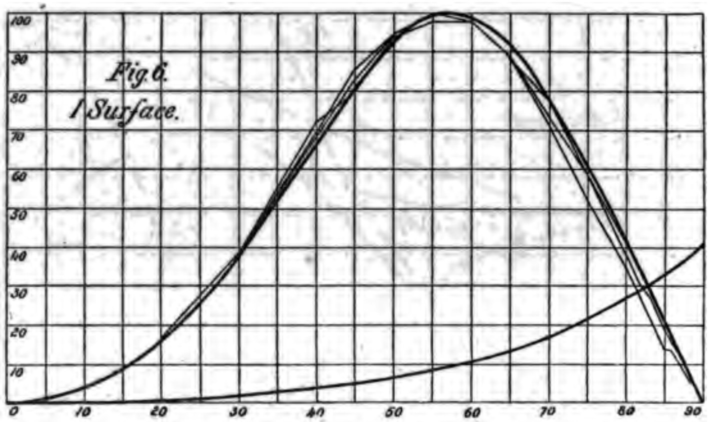


Fig. 6. 1 Surface.

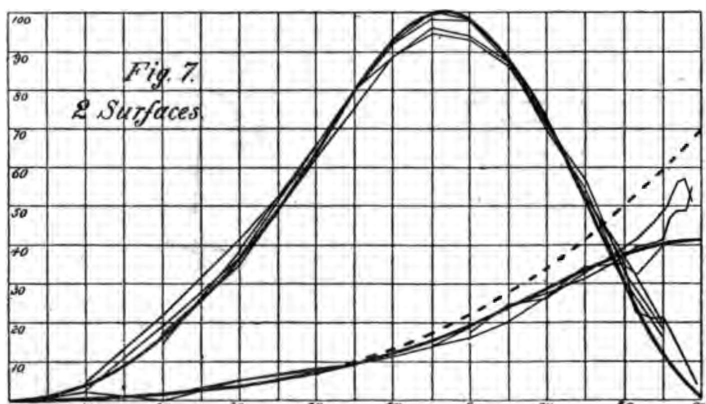


Fig. 7. 2 Surfaces.

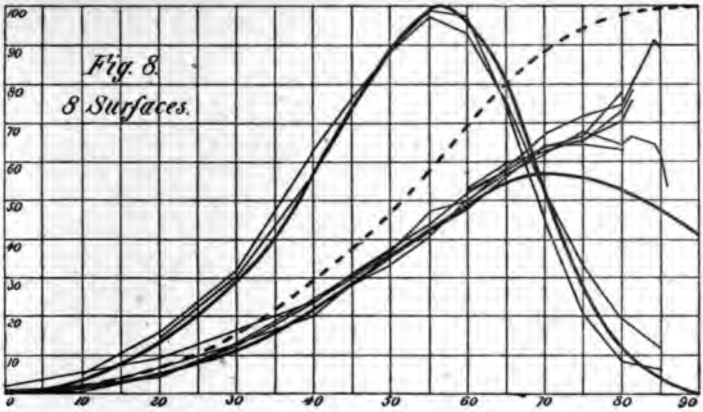


Fig. 8.
8 Surfaces.

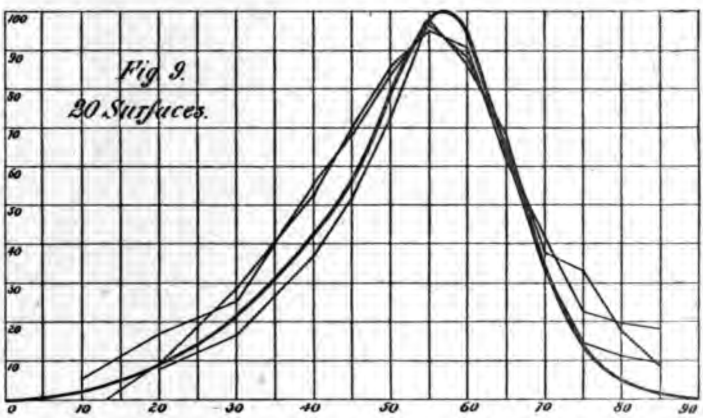


Fig. 9.
20 Surfaces.

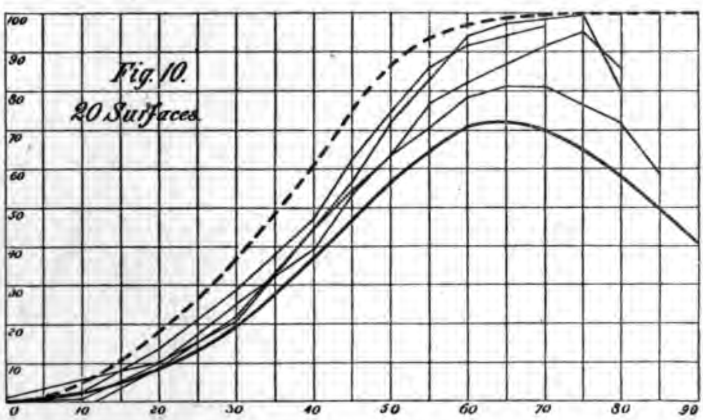


Fig. 10.
20 Surfaces.

XI.

APPLICATIONS OF THE GRAPHICAL METHOD.

By Prof. Edward C. Pickering.

Read, May 12, 1874.

To establish any physical law, or the relation between two quantities so connected that a change in one produced a corresponding variation in the other, the following method is commonly adopted. A great many corresponding values of the two variables are determined with the greatest possible care, and corrected for all the errors to which they are known to be subject. Other errors known as accidental errors, however, still remain, whose minuteness depends on the excellence of the method of measurement and the care exercised. If now an equation can be found to satisfy all these values, its expression in common language will be the required law. Two methods are in common use to determine this equation. First, analytically, the form of the equation is assumed, and the values of the constants in it are found, by assuming certain of the observations to be correct; or, better, by the theory of probabilities employing all the observations, and computing from them the most probable values of these constants. The principal objection to this method is that it furnishes no means of discriminating between the accidental errors and the real variations from the law. In the second or Graphical Method, the two variables are taken as co-ordinates, and points are constructed corresponding to each observation. A curve is then drawn, coinciding with them as nearly as possible, and its equation determined by trial. The form of the curve shows very clearly the accidental errors or other causes of deviation; and the want of accuracy of this method, when the accidental errors are only small, may be completely overcome by the method of residual curves (Journal Franklin Instit., April, 1871).

There still remains, though in a less degree than in the Analytical Method, the difficulty of deciding whether a variation is due to errors, or to incorrect value of some of the constants taken. And it is to meet this difficulty that the following method is proposed.

With the exception perhaps of the circle, the straight line is the only curve of whose correctness every one is a judge; if, then, by any device we can so transform a curve that it shall become a straight line, a moment's inspection will show whether the agreement with observation is real.

Let us first take a special case, and then proceed to the more general discussion. A great many physical laws may be expressed by the equation $y = m x^n$, or that one quantity, y, is always proportional to some power of the other, x. For example, the variation of gravity, of the intensity of light, heat, and electric attraction, with the distance, may be stated $y = m x^{-2}$, or $n = -2$. For elastic forces, $n = 1$, or is proportional to the distance; for Mariotte's law, $n = -1$; for the deflection of a beam in terms of its length, $n = 3$, and so on. Sometimes n is a fractional number, or has a much larger value; thus Wertheim suggests that the laws of elasticity may be explained by assuming that the force of attraction of the particles varies at the 14th power of their distance apart. In the same way, Rankine adopts the exponents $n = -\frac{17}{16}$ and $n = -\frac{10}{9}$ for the variations of the pressure and volume of steam in the cylinder of an engine. Suppose now that we have a number of points constructed, and wish to see if they can be represented by any curve of the form $y = m x^n$. By drawing curves taking various values of n, as 1, 2, 3, $\frac{1}{2}$, &c., we may find one which will agree, but it will be difficult to be sure whether some other value will not give a more exact concordance. If, however, we take logarithms of both sides, and write $\log y = \log m + n \log n$, and calling $\log y = Y$, $\log n = X$, and $\log m = M$, construct a curve with Y and X as coordinates, we obtain $Y = M + n X$. If now the result is a straight line, or differs from a straight line only by the accidental errors, — that is, if there is no curvature to one side more than on the other, — we know that y varies as some power of x, and the value of n is readily determined from the tangent of the angle the line makes with the axis of X. In the same way m is obtained by finding the number whose logarithm is M, the ordinate of the point where the line meets the axis of Y. On the other hand, if the line is not straight, but curved, we may be sure that there is no value of n which will satisfy the observations, or that y is not proportional to *any* power of x. Let us next see how far this method may be generalized. In the first place, instead of x and y we may use any functions f and f' which include only x, y, and *known* constants; that is, which do not include m or n. For example, the equation $y^3 = m x^4 + n x^2$ may be written $\frac{y^3}{x^2} = m x^2 + n$,

or calling $Y = \frac{y^3}{x^2}$ and $X = x^2$, we have an equation of a linear form, $Y = m X + n$. We may then in general write: —

$$f = m f' + n \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

And, if one equation can be reduced to this form, we can readily determine whether there are any values of m and n which will satisfy it. Again, if we have: —

$$f = m f'^{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

we can reduce to a linear form by using $Y = \log f$ and $X = \log f'$. Another common case is: —

$$f = m\, n^{f'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Taking logarithms $\log f = \log m + f' \log n$, from which $\log m$ and $\log n$, and hence m and n, are readily found. The most important application of this formula is when two quantities are so connected that if one varies in arithmetical progression, the other will vary geometrically. This is the case for the variations of the barometer for various heights, for the conduction of heat, and the loss of potential of an insulated cable by leakage. In all these cases we have $y = m\, n^{x}$, where x varies arithmetically and y geometrically, and from which m and n may be determined as above.

Probably the best way of illustrating these principles is by a few examples, in which, however, figures would be required to show the results most clearly.

1. *Torsion Pendulum.* Four observations were made on the time of vibration of a torsion pendulum when its length was varied. The results are given in columns 1 and 2 of the adjoining table. The 3d

TABLE I.

Length.	Time.	Log l.	Log t.	T.
1201.	6.3	3.08	.80	.80
949.	5.7	2.98	.76	.75
706.	4.8	2.85	.68	.68
445.	3.8	2.68	.58	.60

and 4th columns give their logarithms. On constructing the points with these co-ordinates, they fall very nearly on a straight line, as is shown by column 5, which gives values of log t, computed by the for-

mula $C = \frac{1}{2} \log l + .75$. The close agreement shows that $t = m\sqrt{l}$, or that the time is proportional to the square root of the length.

2. *Force of Magnetism.* The observations given in Table II. were made by Professor Mayer (Amer. Jour. Sci., Sept. 1870), to determine the effect of a coil on a galvanometer needle placed at different distances.

TABLE II.

D.	Log *D.*	Log *f.*	*C.*
4	.602	.897	.900
5	.699	.627	.630
6	.778	.413	.413
7	.845	.234	.230
8	.903	.072	.071

The first column gives the distance, the second its logarithm, and the third the logarithm of the force produced, or the tangent of the angle of deflection. To see if $f = m\, d^n$, a curve was constructed, with columns 2 and 3 as co-ordinates, and appeared to coincide very closely with the line $y = -2.76\, x + 2.545$. Column 4 gives the values of $\log f$ thus computed, which shows a close agreement with observation. The result found by Professor Mayer was $n = 2.7404$; but the last two figures should be omitted, as they alter the result by only about one or two hundredths as much as the accidental errors.

3. *Resistance of Air.* Another excellent example is found in the resistance of air to projectiles. Newton assumed that the resistance was proportional to the square of the velocity, or $R = m\, v^2$; but this result is not sustained experimentally. The agreement with the cube of the velocity is, in fact, more exact; but neither is the true law. A more careful examination shows that the law alters for velocities above and below that of sound, or about 1,100 feet per second; since above that velocity the air cannot flow in rapidly enough to fill the space behind the shot, but leaves a vacuum. To show this, a series of observations with the Bashforth chronograph were examined, and showed in a marked manner the change when $i = 1{,}100$. No part of the curve, however, for either spherical or elongated shot becomes a straight line; and therefore no power of the velocity will give the correct value of the resistance.

4. *Conic Sections.* It often happens, especially in astronomy, that

having a number of points we wish to see if they lie on any curve of the second degree. For instance, suppose the polar co-ordinates of the various points given, with the pole at the focus: then $r = \frac{m}{1 + n \cos v}$, in which we wish to see if any values of m and n satisfy all the conditions. The equation may be written $\frac{1}{r} = \frac{1}{m} + \frac{n}{m} \cos v$, which becomes linear if $\frac{1}{r} = Y$, and $\cos v = X$. In the same way, if referred to its centre, $\frac{x^2}{m^2} + \frac{y^2}{n^2} = 1$, make $X = x^2$ and $Y = y^2$, when $\frac{1}{m^2}$ and $\frac{1}{n^2}$ are obtained.

5. *Periodic Functions.* In the study of periodic functions the equation $y = m \sin (n x + p)$ is assumed, in which m determines the maximum amplitude, n the period, and p the phase. If m is given $= a$, n and p may be found by writing $n x + p = \sin^{-1} \left(\frac{y}{a}\right)$, and using as co-ordinates x and $\sin^{-1} \left(\frac{y}{a}\right)$. If the period is given, or $n = b$, we have $y = m \sin b x \cos p + m \cos b x \sin p$, or, dividing by $\cos p$, $\frac{y}{\cos b x} = m \cos p \text{ tang } b x + m \sin p$, in which we may make $\frac{y}{\cos b x} = Y$, $\text{tang } b x = X$, and thus determine $m' = m \cos p$ and $n' = m \sin p$. From these two equations we finally obtain $\text{tang } p = \frac{n'}{m'}$ and $m = \sqrt{n'^2 + m'^2}$.

6. *Lissajous' Curves.* The wonderful variety of curves obtained by Lissajous, by mirrors attached to tuning-forks, may all be reduced to straight lines by this method. They may all be represented by the equations $x = \sin v$ and $y = \sin (m v + n)$, in which m represents the interval of the forks, and n the difference of phase. Eliminating v, we have $m \sin^{-1} x + n = \sin^{-1} y$, which at once takes the linear form when the co-ordinates $X = \sin^{-1} x$ and $Y = \sin^{-1} y$ are employed. To test this, a curve was drawn with an instrument devised by the writer (Journ. Frank. Inst., Jan. 1869), and forty-eight points on it measured, corresponding to variations of v of 30°. In the following table a portion only of the results are given. The first column gives various values of v, the second the measured value of the arc whose sine is y, or of $m v + n$. Constructing a curve with these co-ordinates, we obtain very nearly a straight line, with equation $y = \frac{3}{4}\frac{4}{5} v + 4°$, from which we infer that the difference in phase of the two forks at 0° was 4°, and their interval a little more than a fourth (3 : 4), $\frac{3}{4}\frac{4}{5} = .755$. The differences, as given in the fourth column, are very small, considering the roughness of the measurements.

TABLE III.

v.	$m\,v + n$.	$\frac{34}{45}\,v + 4^{\circ}$.	D.
0	0.	4.	—4.
180	139.	140.	—1.
360	276.	276.	0.
540	412.	412.	0.
720	544.	548.	—4.
900	685.	684.	+1.
1080	817.	820.	—3.

But it is needless to multiply these examples further. The method, in combination with that of residual curves, may be applied to almost any case where an empirical formula is to be deduced, and enables us to find the best values of two of the constants. A wide field seems to be open for it in astronomy; and it is to be regretted that heretofore astronomers should have neglected the graphical methods of discussing errors, as much as physicists have overlooked the more rigid analytical methods.

III.

GRAPHICAL INTEGRATION.

BY EDWARD C. PICKERING.

Presented, Oct. 13, 1874.

WHEN determining the relation between two physical quantities, we sometimes are able to measure only the relative rates at which they alter, instead of the alterations themselves. Or, to speak mathematically, if $y = f(x)$, instead of measuring various corresponding values of x and y, we can obtain only the values of x and $\frac{dy}{dx} = f'(x)$. Of course, if the form of $f'(x)$ is known, the ordinary methods of integration give $f(x)$ and y. But in general this is not given, and the usual methods of approximation are liable to introduce large errors, since by the summation the error adds, and the deviation continually becomes greater and greater. The problem is perhaps better understood by some familiar examples. Thus, given the velocity of the wind at certain times, to determine its total distance travelled per hour; given the velocity of a river, at various points of its cross-section, to find its total discharge; given the strength of an electric current, to find the total quantity transmitted. The case which actually suggested this problem was in calibrating a thermometer tube, having given the length of a mercury column at various points in the tube, to determine the correction to be applied for unequal diameters of the tube at various points. Here the various lengths of the column give the values of $\frac{dy}{dx}$, and the distance of its centre from one end gives the corresponding values of x. Construct a curve with these two quantities as coordinates, and the area included between this curve and the axis of X serves to measure the true values of y. To determine this area, draw a number of equidistant ordinates, and read from the curve the length of each. Then compute by Simpson's formula, $A = \frac{1}{3} a [(y_0 + 4 y_1 + y_2) + (y_2 + 4 y_3 + y_4) + \&c.]$, the area included between

each second ordinate, the curve, and the axes. It gives the ordinate of points of the required curve, $y = f(x)$, the abscissa being of course that of the limiting ordinate.

To test these principles by an actual example, the following method was employed. A smooth curve was drawn by a pencil on a sheet of paper divided into squares, and the co-ordinates of six points on it noted as follows: $x = 0.7$, $z = .84$; $x = 2.3$, $z = 1.14$; $x = 4.4$, $z = 1.65$; $x = 5.8$, $z = 2.05$; $x = 7.6$, $z = 2.69$; $x = 9.6$, $z = 3.54$. It was then assumed that by some measurement these observations had been obtained, and that while x represented one of the variables z gave the relative rate of change, or $\frac{dy}{dx}$. These points were then laid off on a fresh piece of paper, and a smooth curve drawn through them. Of course this should agree with the original curve, were there no errors and the deviation serves to show the amount of error to be expected. To obtain two independent results, a third curve was constructed, like the second, on another piece of paper. The values of z for $x = 1, 2, 3$, &c., were then determined on curves two and three, with results given in Table I., columns two and three. Applying Simpson's formula gives the numbers in columns four and five, which it will be seen agree very nearly, the difference being but little more than the errors of observation. Of course, if necessary, still closer results could be obtained, by residual curves and other methods; but in general the accidental errors present in the original observations render this refinement unnecessary.

TABLE I.

x	z'	z''	y'	y''	Δ
0	.73	.72	0.000	0.000	.000
1	.87	.87			
2	1.08	1.06	.882	.877	—.005
3	1.30	1.30			
4	1.54	1.54	2.185	2.177	—.008
5	1.82	1.82			
6	2.12	2.11	4.075	4.065	—.010
7	2.46	2.47			
8	2.84	2.82	6.542	6.533	—.009
9	3.26	3.26			
10	3.77	3.77	9.816	9.805	—.009

As another example, suppose the following measurements made in calibrating a thermometer tube: $x = 5°$, $z = 10°.0$; $x = 28°$, $z = 10°.4$; $x = 54°$, $z = 10°.7$; $x = 83°$, $z = 10°.9$, in which z gives the length of the mercury column, and x the position of its middle point.

The problem is to determine the correction to be applied to the observed temperatures, assuming the 0° and 100° points to be correct. Constructing a curve with the co-ordinates given above, we deduce the points given in columns one or two of Table II. Now, calling m the volume of the mercury drop, we have $z : m = dx : dv$, or $\frac{dv}{dx} = \frac{m}{z}$. Hence, we must use for ordinates in our summation the reciprocal of z as given in column three. Treating these as before, we obtain by the formula column four, and dividing by the total sum 283.8 gives in column five the true temperature, and subtracting the observed readings from these gives the correction in column six.

TABLE II.

t°	L	$\frac{dv}{dx}$	v	t'°	Δ
0	9.9	10.10	0.0	0°.00	0°.00
10	10.1	9.90			
20	10.3	9.71	59.4	20°.93	0°.93
30	10.4	9.62			
40	10.5	9.52	117.1	41°.26	1°.26
50	10.6	9.43			
60	10.7	9.35	173.3	61°.06	1°.06
70	10.8	9.26			
80	10.9	9.17	228.9	80°.65	0°.65
90	10.9	9 17			
100	11.0	9.09	283.8	100°.00	0°.00

To determine how rapidly the errors diminish, increasing the number of ordinates, the area included between the axis and the curve $y = \frac{1}{\pi} \sin x$ was computed for 2, 4, 6, 12, and 18 divisions; the errors in these cases were .030047, .001454, .000276, .000019, .000003, so that a high degree of accuracy is readily obtained. M. Chevilliet has recently shown (*Comptes Rendus*, lxxviii. p. 1841) that the error in Simpson's formula depends on $\frac{h^4}{180} \frac{d^3 y}{d x^3}$, while the method of summing by trapeziums gave $\frac{h^2}{12} \frac{dy}{dx}$. In an example he finds that the area of the curve $x \log x$, between $x = 10$ and $x = 20$, is given correctly by Simpson's formula, taking ten intervals, within .000005, while by the method of trapezoids the error is .001809. Evidently, then, it is easy to obtain by the first of these formulas as great an accuracy in the result as is needed in almost any physical research.

VII.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

No. I.

FOCI OF LENSES PLACED OBLIQUELY.

BY PROF. E. C. PICKERING AND DR. CHAS. H. WILLIAMS.

Presented, Feb. 9, 1875.

THE following experiments were suggested by noticing that the spot of light of a reflecting galvanometer was thrown out of focus when the mirror turned slightly. The image in this case was formed by a lens near the mirror; and, to obtain a distinct image, it was found that the ends of the scale must be brought much nearer the mirror, owing to the obliquity of the rays upon the lens. It was further noticed that the focus was greatly altered when the slit used was placed horizontally instead of vertically. To study the matter more carefully, the following apparatus was constructed.

To one end of a board about five feet long was fitted a small telescope; ten inches from this was placed a graduated circle, which rested horizontally on the board; and at its axis was fixed a screen of sheet iron, which stood vertically and had a hole in the centre, on a level with the telescope.

On one side of the screen, and covering the hole, was fixed a biconvex lens of 25.5 inches focus, so placed that, when the graduated circle was moved, the lens turned about a vertical axis which coincided with that of the circle. At the farther end of the board was placed a small gas flame, and between the lens and gas was a screen which moved over a scale giving the distance in inches from the axis of the lens. At the centre of this screen, on a level with the lens, telescope, and gas flame, and in the same straight line, two fine slits were cut, one vertical the other horizontal, intersecting each other in the middle; and in these slits filaments of silk were stretched lengthwise, to aid in focussing.

To use the instrument, the gas was lighted, then the screen with the cross-hairs was placed at the principal focus of the lens: that is, 25.5

inches from it. The graduated circle, carrying the lens, was now brought to zero, so that the rays from the cross should fall normally on the centre of the lens. Lastly, the telescope was focussed on the cross-hairs. The zero point of the graduated circle was obtained with great accuracy, by lighting the gas, then, placing the eye at some distance behind the gas flame, the graduated circle was moved till the reflections from the anterior and posterior surfaces of the lens exactly coincided with the flame; in this way the true vertical as well as horizontal position was obtained. The instrument being thus adjusted, the graduated circle and lens were turned five degrees. The screen having the cross slits was now moved, by means of a rod attached to it, until the rays from the vertical slit were properly focussed by the observing telescope, the distance of the screen from the lens was read from the scale, the reading repeated three times, and the mean recorded. The same was afterward done for the rays from the horizontal slit. The graduated circle was then moved on, and the same readings repeated every five degrees. It was impossible to take readings from the vertical slit beyond 65°; for, after that, the screen could not be brought near enough to the lens to focus the rays properly, and the image became quite indistinct; but with the horizontal slit the readings were continued to 85°. After completing this set of readings, the screen was placed at one and a half times its focal length from the lens, the graduated circle brought to zero, and the telescope focussed as before; then the same readings were repeated every five degrees, also when the screen was at one half and at twice the focal distance.

Having obtained these readings, curves were constructed by the Graphical Method, the vertical distances being equal to the distance from screen to lens, and the horizontal to the angle through which the graduated circle was moved. As a test for the accuracy of the readings when the telescope was focussed for different points, all the readings were reduced, so as to be compared with those taken when the distance from lens to screen was equal to the focal length of the lens. This was done by means of the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, in which u and v are the conjugate foci, and f the principal focus of the lens. In these experiments, $\frac{1}{u}$ is a constant, for, when the telescope has been once focussed, it remains fixed through that set of readings, and the reciprocal is easily found by $\frac{1}{u}=\frac{1}{f}-\frac{1}{v}$; that is, subtracting the reciprocal of the distance from screen to lens, when the angle is equal to zero, from the reciprocal of the principal focus of the lens. This reciprocal is to be

added to those of the readings, and thus the readings of any set are rendered equivalent to those taken when the screen is at a distance from the lens equal to its principal focus. This being done, the greatest variation of any of the readings from the standard was found to be a little over one per cent.

The result of these measurements of the vertical slit are given in Table I., and of the horizontal slit in Table II. Column 1 gives the

TABLE I.

i.	.5*f.*	*f.*	1.5*f.*	2*f.*	*f.*	.5*f.*	1.5*f.*	2*f.*	Mean.
0°	12.7	25.5	88.2	51.0	25.5	25.5	25 5	25.5	25.50
5°	12.6	25.3	37.6	49.3	25.3	25.1	25.2	25.1	25.17
10°	12.8	24.3	35.6	46.5	24.3	23.9	24.3	24.3	24.20
15°	12.0	23.0	33.0	41.0	23.0	22.8	23.1	22.8	22 92
20°	11.5	21.9	30.1	37.5	21.9	21.1	21.6	21.6	21.55
25°	11.0	19.8	26.3	32 6	19.8	19.5	19.6	19.9	19.70
30°	10.2	17.7	23.0	27.0	17.7	17.1	17.7	17.7	17.55
35°	9.2	15.1	19.1	22.0	15.1	14.5	15.3	15.4	15.07
40°	8.1	12.8	15.5	17.0	12.8	11.9	12.9	12.8	12.60
45°	7.0	10.8	12.1	13.0	10.3	9.6	10.5	10.4	10.20
50°	5.7	8.1	9.2	9.5	8.1	7.3	8.2	8.0	7.90
55°	4.7	6.1	6.5	7.0	6.1	5.7	5.9	6.1	5.95
60°	3.7	4.3	4.4	4.7	4.3	4.3	4.1	4.3	4.25
65°	2.6	3.1	3.1	3.2	3.1	2.9	2.9	3.0	2.97
70°									
75°									
80°									
85°									

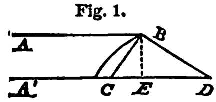
TABLE II.

i.	.5*f.*	*f.*	1.5*f.*	2*f.*	*f.*	.5*f.*	1.5*f.*	2*f.*	Mean.
0°	12.7	25.5	38.2	51.0	25.5	25 5	25.5	25.5	25.50
5°	12.8	25.4	38.1	50.5	25.4	25.9	25.5	25.4	25 55
10°	12.7	25.4	37.9	49.7	25.4	25.5	25.4	25.2	25.37
15°	12.7	25.0	37.1	48.9	25.0	25.5	25.0	25.0	25.12
20°	12.5	24.6	36.2	47.0	24.6	24.7	24.6	24.5	24.60
25°	12.4	24.0	35.1	44.7	24.0	24.3	24.1	23.8	24.05
30°	12.2	23.3	34.0	43.4	23.3	23.6	23 6	23.4	23.47
35°	12.0	22.5	32.2	41.4	22.5	22.8	22 7	22.8	22.70
40°	11.8	21.9	30.5	37.9	21.9	22.1	21.8	21.7	21.87
45°	11.5	20.9	28.9	34.7	20.9	21.1	21.0	20.7	20.92
50°	11.2	19.7	26.7	32.4	19.7	20.1	19.8	19.8	19.85
55°	10.9	18.4	25.2	30.0	18.4	19.1	19 0	18.9	18.85
60°	10.5	17.5	23.5	27.6	17.5	17.9	18.0	17.9	17.82
65°	10.1	16.6	21.5	24.7	16.6	16.8	16.8	16.7	16.72
70°	9.8	15.7	20.0	22.7	15.7	16.0	15.9	15.7	15.82
75°	9.5	14.8	18.2	20.5	14.8	15.2	14.7	14.6	14.82
80°	9.0	13.7	16.8	18.7	13.7	14.0	13.8	13.7	13.80
85°	8.6	12.8	15.3	17.0	12.8	13.0	12.8	12.8	12.85

angle of incidence, the next four columns the observed conjugate focus, u, or position of the slit when the telescope was focussed on a point seen through the lens at a distance of $.5f$, f, $1.5f$, and $2f$, in turn. The next four columns give the computed value of f, assuming that a lens placed obliquely conforms to the law $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, as well as when in the ordinary position. The result justifies this assumption; for the four values of f are nearly coincident, and agree well with the mean given in the last column. The phenomena are thus greatly simplified, since we have now only to consider the case of the principal focal distances, or that the incident ray forms a parallel beam.

To represent these results theoretically, let us suppose the slits and lens so small, compared with their distance apart, that we may neglect all aberration except that due to the obliquity of the incidence. Considering first the case of the vertical slit, let Fig. 1 represent the section of a horizontal plane passing through the centre of the lens. Then let D represent the position of the slit when the emergent rays are parallel; that is, when AB is parallel to $A'C$. Now $CD=f'$ is the new focal length which is to be determined. Call f the principal focal distance, n the index of refraction, i and r the angles of incidence and refraction of the light on entering the lens, and r' and i' the corresponding angles on its emergence. Call also A the angle between the two surfaces of the lens at its edge, or of the two surfaces where pierced by the ray. Then, by the law of refraction, $\sin i = n \sin r$, and $\sin i' = n \sin r' = n \sin (r + A) = n \sin r + n A \cos r = \sin i + n A \cos r$, since $r' = A + r$ and $\sin A$ being very small may be regarded as equal to A. Again, $\sin i' - \sin i = \cos i\ (i' - i) = n A \cos r$, and hence, $\frac{i'-i}{A}=\frac{-n \cos r}{\cos i}$.

Fig. 1.



Now, in the triangle BCD we have $BDC=i-i'-A$, $BCD=90-i$, and BD sensibly equal to f'. Again, $BC=fA\ (n-1)$, for by the formula for lenses $\frac{1}{f}=(n-1)\left(\frac{1}{R}+\frac{1}{R'}\right)$, or $f=\frac{R}{2\,(n-1)}$, but $A=\frac{2\,BC}{R}$ $\therefore f=\frac{BC}{A\,(n-1)}$, or $BC=fA\,(n-1)$. Since the sides are proportional to the sines of the opposite angles $BD:BC=\sin BCD : \sin BDC$, or $f':fA\,(n-1)=\sin(90-i):i-i'-A$ or $f'=f.\frac{A\,(n-1)\cos i}{i-i'-A}$; dividing by A, and substituting the value of $\frac{i-i'}{A}$ given above, we have $f'=f\,\frac{(n-1)\cos^2 i}{n\cos r-\cos i}=f.\,\frac{(n-1)\cos^2 i}{\sqrt{n^2-\sin^2 i}-\cos i}=f.$

$\frac{(n-1)\cos^2 i}{n^2-1}(\sqrt{n^2-\sin^2 i}+\cos i)=f\,\frac{\cos^2 i}{n-1}(\sqrt{n^2-\sin^2 i}+\cos i).$

By means of this formula, the value of f' was computed for every 5° for $n=1.5$; and the results, calling $f=100$, are given in column 2 of Table III. Column 1 of the same table gives the corresponding angle of incidence, and column 3 the rate of change of f for small changes of n, or $\frac{df}{dn}$. This is only serviceable if we wish to compute the foci of lenses of various indices, but it is applicable only for value of n near 1.5 or 1.6. As an example, suppose we wish the focus of a lens having an index of refraction $=1.57$, and inclined 45°. Then $f'=40.6+.07\times.6=41.0$. The values in column 4 are computed in this way, and give the foci for the lens actually employed, whose index was assumed to be equal to 1.55. To compare these results with observation, the last column of Table I. was reduced by dividing by 25.5, the principal focal distance. The differences or errors are given in column 6, which show the close agreement with theory. From these it appears that the deviations are probably mainly due to accidental errors, the preponderance of negative values rendering it probable that the focal distance 25.5 was taken as too small.

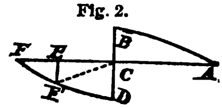
TABLE III.

i.	1.5.	$\frac{df}{dn}$	1.55.	Obs.	ε.
0°	100.0	0.0	100.0	100.0	0.0
5°	98.9	0.0	98.9	98.7	−0.2
10°	96.1	0.1	96.1	94.9	−1.2
15°	91.2	0.2	91.3	89.8	−1.5
20°	84.8	0.2	84.9	84.5	−0.4
25°	77.0	0.3	77.1	77.2	+0.1
30°	68.4	0.4	68.6	68.8	+0.2
35°	59.2	0.5	59.4	59.1	−0.3
40°	49.8	0.6	50.1	50.3	+0.2
45°	40.6	0.6	40.9	40.0	−0.9
50°	32.0	0.6	32.3	31.0	−1.3
55°	24.1	0.6	24.4	23.3	−1.1
60°	17.1	0.5	17.3	16.7	−0.6
65°	11.3	0.4	11.5	11.6	+0.1
70°	7.0	0.3	7.1		
75°	3.8	0.2	4.0		
80°	1.6	0.1	1.6		
85°	0.4	0.0	.4		
90°	0.0	0.0	0.0		

The case of the horizontal slit is more complicated, since the rays no longer remain in one plane. Considering only those rays in the vertical plane passing through the axis around which the lens turns,

and one point of the slit, we see that they will strike the lens at an angle of incidence about equal to i, will traverse it in a plane which we will call the plane of refraction inclined to the first plane $i - r$, and finally emerge in a plane parallel to the first. The plane of refraction will intersect the lens along two circles whose distance apart at the centre will be greater than the thickness of the lens in the ratio of $\cos r$ to 1; hence their radii R' will be less than the radius of curvature R of the surfaces of the glass in the same ratio, or $R' = R \cos r$. Again, the apparent index of refraction n' will be different, and since $\frac{1}{f} = \frac{2(n-1)}{R}$ and $\frac{1}{f'} = \frac{2(n'-1)}{R'}$, we have $f' = f\frac{n-1}{n'-1} \cdot \frac{R'}{R} = f \cos r \frac{n-1}{n'-1}$. It therefore only remains to determine n', the apparent index of refraction. As the problem is one in spherical trigonometry, suppose a sphere described around the centre of the lens and projected in Fig. 2, the eye lying in the axis of the lens prolonged. Let $CA = i$, the angle through which the lens has been turned, and $CE = r$, the corresponding angle of refraction. Then if the surface of the glass is vertical, as at the centre of the lens, the incident ray will be AC and the refracted ray CE.

Fig. 2.



Next suppose the surface slightly inclined by the amount $CD = BC = v$, as is the case for the upper and lower parts of the lens. $AB = i'$ will now be the angle of incidence; and, to construct the refracted ray, we have first the condition that $\sin i' = n \sin r'$, and secondly that the incident and refracted ray shall lie in the same plane with the normal BCD. To construct it, pass a plane through the normal BC and the incident ray AC, which will intersect the sphere along the great circle AB and FD; on this, lay off $DE' = r'$ such that $n \sin r' = \sin i'$, but, as v is infinitesimal, i' will be sensibly equal to i, and r' to r. Now in the right-angled spherical triangle FCD, $\sin CD = \sin DF \sin CFD$, or $\sin v = v = \sin i \times F$, or $F = \frac{v}{\sin i}$; and in the triangle FEE', $\sin EE' = \sin FE' \sin EFE'$, or $EE' = \sin (i - r) F$, or, substituting the value of F just found, $EE' = v \frac{\sin (i-r)}{\sin i}$. Calling i'' and r'' the angles of incidence and refraction of the ray with regard to the section of the lens made by the plane of refraction, then i'' will not equal BC, but will be the angle which, when projected on the plane of the section of the lens, will be BC or $i'' \cos r = BC \therefore i'' = \frac{v}{\cos r}$. Again EE' is the angle

through which the ray is bent, or $i'' - r'' = v \frac{\sin(i-r)}{\sin i}$, or subtracting, $r'' = v\left[\frac{1}{\cos r} - \frac{\sin(i-r)}{\sin i}\right] = v \frac{\sin i - \sin(i-r)\cos r}{\sin i \cos r}$; but $n' = \frac{i''}{r''}$, which, with the above values, gives $n' = \frac{\sin i}{\sin i - \cos r \sin(i-r)}$. Substituting this value in the equation $f' = f \cos r \frac{(n-1)}{(n'-1)}$ gives $f' = f (n'-1) \frac{\sin i - \sin(i-r)\cos r}{\sin(i-r)}$. This would be the focal distance if the rays on emerging remained in the plane of the section of the lens. But they pass into a plane inclined to this $i - r$, hence the observed focus f'' will be such that when projected on the plane of the section it will equal f', or $f'' \cos(i-r) = f'$. Hence finally $f'' = f (n-1) \frac{\sin i - \sin(i-r)\cos r}{\sin(i-r)\cos(i-r)}$. This last step may be open to criticism, but the close agreement with observation seems to justify it.

In Table IV., this formula is compared with observation, as the law for the vertical slit is compared in Table III. The columns in the two tables correspond, and it will be noticed that the agreement is very close.

TABLE IV.

i.	1.5.	$\frac{df}{dn}$	1.55.	Obs.	ε.
0°	100.0	0 0	100.0	100.0	0.0
5°	99.9	0.0	99.9	100.2	+0.3
10°	99.2	0.2	99.3	99.5	+0.2
15°	97.7	0.3	97.8	98.5	+0.7
20°	96.1	0.5	96.3	96.4	+0.1
25°	93.7	0.6	94.2	94.3	+0.1
30°	91.1	0.8	91.5	92.0	+0.5
35°	88.3	1.0	88.8	89.0	+0.2
40°	84.7	1.2	85.3	85.7	+0.4
45°	81.1	1.5	81.8	82.0	+0.2
50°	77.2	1.7	78.0	77.8	—0.2
55°	73.2	1.9	74.1	73 9	—0.2
60°	69.0	2.1	70.0	69.8	—0.2
65°	64.7	2.4	65.9	65.5	—0.4
70°	60.4	2.6	61.7	62.0	+0.3
75°	56.3	2.8	57.7	58.1	+0.4
80°	52.1	3.0	53.6	54.1	+0.5
85°	48.3	3.2	49.9	50.4	+0 5
90°	44.8	3.4	46.5		

The principal practical application of these results is to photographic lenses. It will be seen that a single lens, even if perfectly corrected for spherical and chromatic aberration, is still subject to this defect. Con-

structing the curves with polar co-ordinates, taking the radius vector equal to the focal length and its angle equal to the angle of incidence, we obtain a line every point of which would be in focus at the same time. This shows that in a photographic camera for lines passing through the axis, corresponding to the vertical slit, the surface instead of being a plane should have a radius of curvature of only .3 the focus. For lines perpendicular to these, or circles concentric with the centre, corresponding to the horizontal slit, the curvature should be .7 the focus. We also see the importance of having telescope lenses carefully centred, and why the images of stars, if this adjustment is not exact, are elliptical instead of circular.

Since writing the above, a further application of these formulas has been suggested in the case of the eye, that the imperfect vision at a distance from the centre of vision may be due to the rays passing obliquely through the lens. It will also be noticed that the curvature of the retina corresponds nearly with that which would give the best vision. As stated above, for radial lines the radius of curvature should be about .3, and for concentric circles .7, its distance from the lens. The actual curvature in the normal eye is about .5, or the mean of these values.

XIII.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

E. C. PICKERING, PROFESSOR OF PHYSICS.

No. II.—LIGHT TRANSMITTED BY ONE OR MORE PLATES OF GLASS.

BY W. W. JACQUES.

Read, April 13, 1875.

THE following experiments were made for the purpose of determining the percentage of light transmitted through 1, 2, ..., 10 plates of glass, normal to the direction of the light, and of one, four, and ten plates when i was 0°, 5°, ..., 65°.

The apparatus used consisted of a triangular frame, isosceles and right angled, having a periphery of 100 inches. A gas jet was placed at the right angle, and two mirrors were so placed at the other angles as to reflect the light from the jet along the hypothenuse, thus giving the effect of two equal sources of light 100 inches apart. The plates of glass were mounted on a graduated circle placed between the jet and one of the mirrors, and the light cut off was measured by a Bunsen disc, movable along the hypothenuse of the triangle. (See "Physical Manipulation," Expt. 67. Pickering.)

The plates used were of common 12 × 18 window glass, and were carefully cleaned with rotten-stone, and then dried by rubbing with chamois skin immediately before each experiment.

The experiments were made in a dark room, whose walls were painted black, and it was found that the reflection from a sheet of paper, or even from the clothes of the observer, was sufficient to prevent the accurate setting of the disc. The following tables give the results of the experiments; each number being the mean of four observations, and the probable error of a single observation being 0.42 of one per cent.

Table I. gives the percentage of light transmitted by 1, 2, ..., 10 plates when $i = 90°$. The first column gives the number of plates, the

TABLE I.

Plates.	Observation.	Theory.	Difference.
0	100	100	.0
1	89.5	89.5	.0
2	81.3	81.1	.2
3	74.7	74.1	.6
4	69.3	68.2	1.1
5	64.0	63.2	.8
6	60.0	58.9	.1
7	55.0	55.1	—.1
8	52.0	51.8	.2
9	48.3	48.8	—.5
10	45.3	46.2	—.9

second gives the observed amount of light transmitted, the third gives the amount transmitted as calculated by the formula $t = \frac{1 - A}{1 + (m - 1) A}$, in which A is the amount reflected from one surface, and m the number of surfaces. The quantity A was determined by solving the equation $t = \frac{1 - A}{1 + A}$, in which t was carefully determined by a considerable number of observations. The fourth column gives the differences between the observed and computed values, which are within the limits of errors of observation. It will thus be seen that these observations give a very accurate proof of the above formula.

TABLE II.

i.	One Plate.		Four Plates.		Ten Plates.	
	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.
0°	89.5	91 2	69.3	71.7	45.3	50.6
5°	89.7	91.2	69.3	71.7	45.0	50.6
10°	89.5	91.1	69.3	71.7	44.5	50.7
15°	89.3	91.1	69.3	71.7	44.5	50.8
20°	89.0	91.1	69.0	71.8	44.7	50.9
25°	89.0	91.1	69.3	71.8	44.7	51.0
30°	88.5	90.9	68.7	71.9	45.0	51.6
35°	87.7	90.6	68.7	72.0	46.7	52.4
40°	87.5	90.0	68.5	72.1	48.3	53.8
45°	86.3	89.4	68.3	72.1	50.0	56.6
50°	85.0	88.6	66.5	71.7	53.0	58.7
55°	83.5	86.0	62.0	70.0	52.5	60.3
58°					48.5	
60°	79.5	83.7	54.5	66.0	44.5	56.4
65°	71.7	79.6	45.7	60.0		

Table II. gives the percentage of light transmitted by 1, 4, and 10 plates for different values of i. The first column gives the values of i; the second, fourth, and sixth columns give the observed amounts of light transmitted; and the third, fifth, and seventh columns give the theoretical amounts. These last were calculated from the formula $t = \frac{1}{2}\left(\frac{1-A}{1+(m-1)A} + \frac{1-B}{1+(m-1)B}\right)$, in which A was determined from the equation $A = \frac{\sin^2(i-r)}{\sin^2(i+r)}$, and B from the equation $B = \frac{\text{tang}^2(i-r)}{\text{tang}^2(i+r)}$, by substituting the proper values for the angles of incidence and refraction, assuming the index of refraction to be 1.55. Constructing the points, with abscissas equal to the angles of incidence and ordinates to the observed amounts of light transmitted, it will be found that they form very smooth curves. But it will be noticed that while they agree in general with the theoretical results, assuming that the light is lost by simple specular reflection, the differences are considerable, showing that we ought not in our calculation to neglect the opacity of the glass, imperfection of the surface, and other sources of error.

From the numbers in this table, we conclude that, while the amount of light transmitted by one plate decreases considerably as i increases, the amount transmitted by four plates is more nearly constant for small angles, and the amount transmitted by ten plates actually increases until i becomes 55°; which facts agree with the conclusions arrived at theoretically by Prof. Pickering. (Proc. Amer. Acad. Vol. IX. p. 6.)

It was impossible to carry these experiments beyond $i = 65°$ with the apparatus employed, because the disc came so near the mirror as to cast a shadow upon itself.

XVII.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

E. C. PICKERING, PROFESSOR OF PHYSICS.

III.—INTENSITY OF TWILIGHT.

BY CHARLES H. WILLIAMS.

Read, May 11, 1875.

DURING the fall and winter of 1874 an attempt was made to measure the amount of light given by the sun when at different distances below the horizon. Days were chosen when the sky was perfectly clear at sunset, though a few observations were made when it was snowy or cloudy.

The instrument used was the photometer first described in the report of the Total Eclipse Expedition for 1870. It consisted of a box about five feet long, eighteen inches high, and twelve inches wide; over the top and sides, which were of light framework, black cloth was stretched; a circular hole, about five inches in diameter, was cut in one end and covered by a Bunsen disk, and a standard candle, in a spring candlestick, was moved along the centre of the box by means of a rod attached to it; the distance from candle to disk being varied at pleasure, and measured by a mms. scale attached to the rod.

It was found inconvenient in practice to be obliged to read the scale at every observation, and the disappearance of the spot could be better watched if the eyes were kept fixed on the disk. An arrangement for automatic registering was therefore added. A piece of sheet-iron connected the candlestick with a rod moving outside the box along its whole length, the iron clasped the rod and was held in place by friction; to the iron was fixed a movable point, which could be pressed into a fillet of paper by means of a string passing from the iron round a pulley at each end of the box. The position of the candle was varied by moving the rod; the point where the observation was taken was marked on the paper, and the distance of the candle from the disk in mms. was read off afterward from a scale.

To use the instrument, a suitable day being taken, the disk was exposed to the horizon a few minutes before sunset, the candle lighted, and placed at about fifty mms. from it. The disk was then watched until it became dark-centred; the distance of the candle from the disk was now adjusted, so that the centre of the disk should nearly disappear, when the time was noted, and observations were then taken every minute till the light became very feeble. It was found impossible to get a perfect disappearance of the spot, owing principally to the difference in color of the two lights, the candle being much more yellow than the sun; a certain neutral shade between the dark and light centre was therefore taken as the point for making the observations. Various attempts were made to get rid of this difference of color, but without success. A cell filled with a solution of sulphate of copper of different strengths was placed on the candle side of the disk, also indigo and other blue solutions; the only effect of these was to give the whole surface of the paper a greenish tint when the candle was brought near, without making the disappearance of the spot more perfect. Disks made of paper of different colors, and sheets of plaster of Paris, made extremely thin by pressing the fluid plaster between sheets of plate glass, were tried with the same results. The best material seemed to be fine white paper painted with spermaceti, except at the centre.

It seemed to make no difference in the relative diminution of the light, whether the observations were taken with a clear horizon or with part cut off by some adjoining building; the readings from the upper part of the building looking over the roofs agreeing very well with those taken below. Having made a number of observations on different days, the instrument was tested to get the probable error of any reading. The photometer was placed in a dark room and a fixed amount of daylight admitted, the candle was moved till the disk

TABLE I.

Distance.	Prob. E.	Percentage.
130 mm.	2. mm.	3
175 "	2.9 "	3
240 "	2.5 "	2
440 "	9.5 "	4
455 "	4.1 "	2
520 "	10. "	4
635 "	8.2 "	3
950 "	8.9 "	2

assumed the neutral tint, and the mean of ten readings taken. The amount of light admitted to the room was then increased, and eight sets of readings thus taken. The preceding table gives, in the first column, the mean distance of the candle from the disk for each series of ten settings; in the second, the probable error for each reading; and in the third, the percentage of error.

It will be seen that the probable error is not large enough to seriously invalidate the results of the observations, as the readings taken by the photometer denoted the distance at which a standard candle should be held from the disk to give a light equal to that from the sun at a given time; it was thought best to reduce those readings to some standard, and compare them with the light given by a standard candle burning at a distance of one metre from the disk. Suppose we wish to reduce a reading of 200 mms. to this standard, C, or 1,000 mms. Let $I =$ actual intensity: —

$$C : I = 200^2 : 1000^2 = \frac{1}{1000^2} : \frac{1}{200^2};$$

whence $$I = C \left(\frac{1000}{200}\right) 2 = 25\,C.$$

In this way Table II. was constructed, giving the actual amount of light, the readings being taken every minute.

On the days represented in the first six columns, the observations were all taken with an unobstructed horizon. On Nov. 6 and 7, part of the sky was shut off by surrounding buildings. Jan. 15, there were a few clouds; and Jan. 3, the whole sky was overcast. Nov. 13, a cell with sulphate of copper solution was placed behind the disk; and on Dec. 31, a cell with solution of indigo was used.

To see whether the light decreased according to any function of the time, curves were constructed, taking for vertical distances the logarithm of the observed reading, and for horizontal distances the minutes after sunset at which the observations were made. The result gave a series of nearly straight lines all running in the same direction. In some of the lines, there was a decided bend in the middle, and traces of this were found in almost all. To make this bending more apparent, a residual curve was constructed; this was obtained by comparing each of the curves with a straight line drawn in their mean direction, and making the ordinates of the desired curve the mean of their differences from this straight line. In this way the deviation of the original curves from the straight line was made quite apparent, though the difference was not originally very great. To find the curve which should represent the diminution of the light for each minute after sun-

TABLE II.

Min'tes after Sunset.	Nov. 12.	Nov. 13.	Dec. 31.	Jan. 1.	Dec. 15.	Dec. 21.	Nov. 6.	Nov. 7.	Jan. 15.	Jan. 3.
1							237	177	27	7
2							204	149	25	6
3							204	135	21	6
4							188	114	19	5
5							177	108	16	4
6							149	98	13	4
7							139	91	12	3
8				164			132	81	10	3
9				144		156	121	74	9	2
10			174	125		141	114	64	7	2
11			161	121		134	100	54	6	1
12			144	106		125	89	43	5	1
13			141	104		118	81	37	4	
14			130	94		102	74	31	3	
15			102	77		83	63	26	3	
16	139		84	76		72	59	24	2	
17	132		76	63		59	48	20		
18	108	119	66	52		52	44	18		
19	98	96	?	41	46	45	34	15		
20	77	96	45	35	45	41	27	11		
21	69	69	35	32	39	35	23	10		
22	51	49	30	26	30	29	18	8		
23	46	?	26	23	26	25	15	6		
24	38	38	20	18	23	23	12	5		
25	31	30	17	15	19	20	10	4		
26	22	24	14	14	15	18	7	3		
27	16	20	11	10	11	14	4	3		
28	?	15	8	9	9	11	3	2		
29	13	12	?	7	8	10	3	2		
30	11	9	6	7	6	7	2	2		
31	9	7		5	5	7	2	1		
32	8	6		4	4	6	1	1		
33	6	5		4	?	5	1	.9		
34	4	4		3	3	4	1	.8		
35	?	3		2		3	.9	.6		
36	3	2		2		3	.7			
37	3	2					.7			
38	3	2								

set, the ordinates of the straight line were obtained, to these were added the ordinates of the residual curve for the same times after sunset, both readings being in logarithms of the original observations; this logarithmic sum was now doubled, the sign changed, and the figures so obtained were used as ordinates for a new curve, the abscissas being the times. From this last curve we obtain directly the logarithm of the number which would represent the proportion of light at any minute as compared with that at sunset, which we call unity.

In the following table the minutes after sunset, or abscissas, and the percentage of light as compared with sunset, or ordinates, are given, from which the last curve was constructed: —

TABLE III.

Minutes after Sunset.	Per cent of Light compared with Sunset.	Minutes after Sunset.	Per cent of Light.
0	1.000	18	.094
1	.950	19	.079
2	.817	20	.064
3	.752	21	.055
4	.655	22	.044
5	.597	23	.038
6	.516	24	.031
7	.466	25	.026
8	.407	26	.021
9	.337	27	.015
10	.290	28	.012
11	.261	29	.010
12	.228	30	.009
13	.200	31	.007
14	.177	32	.006
15	.143	33	.005
16	.128	34	.004
17	.101		

IV.—LIGHT OF THE SKY.

By W. O. Crosby.

The light of the sky is reflected light, of which the sun is the source.

It is well known that the light of the sky diminishes as the angular distance from the sun increases. And the following observations were made with a view, first, to determine the absolute amount of light received from the sky at different distances from the sun; secondly, to ascertain the law of the diminution of the light with increasing angular distance from the sun. The apparatus employed consisted of a common mirror, so arranged as to reflect the light horizontally into a darkened room, a condensing lens having an aperture of 9 cm. and a focal distance of 225 cm., and a photometer similar to that employed by Dr. Williams in his observations on twilight.

The method pursued was to so adjust the mirror and lens that an image of the sun would fall upon the disk of the photometer, and then

to measure the intensity of the light at regular intervals of time as the sun receded from the portion of sky from which the light was received.

By allowing the mirror to remain fixed during an entire series of observations, absolute uniformity in the angle of incidence of the light on the mirror was obtained, and thus the percentage of the light reflected by the miror rendered constant, and exact measurements of the angular distance from the sun could be easily made by simply noting the lapse of time.

One result of this method is that all the observations are made east of the sun; the part of the sky from which the light is received being necessarily on the sun's path. A reverse series can be readily obtained on the west side of the sun, by adjusting the mirror so as to receive light from a point at a convenient distance west of the sun, on the path of the same, and then making observations at regular intervals as the sun approaches the point.

The light was received from a circular sky area, 2° 20′ in diameter, or 4.25 square degrees; and the proportion of the light lost by reflection from the mirror, and transmission through the lens, was about .40 of the whole. The following table embraces the results of four series of observations. Column one gives the angular distance from the sun. The second, third, fourth, and fifth columns give the intensity of the light received from one square degree of sky; the unit of intensity being the light of a standard candle at a distance of one metre. Series I. was made Jan. 20, 1875, between the hours of 2.45 and 3.47 P.M., beginning 15° 45′ west of the sun. The declination of the sun at this time was 20°; and, since the angles given in the table for this series were not measured on a great circle, they should be reduced in the proportion of the radius to the cosine of the declination. Series II., III., and IV. were made east of the sun between 12 M. and 1 P.M.; the II. on March 23, the III. and IV. on March 27.

On Jan. 20, the sky was hazy, had a whitish-gray aspect, and reflected much light; on the 23d of March it was clear and blue; and on the 27th very clear, reflecting but little light. The great difference in the intensity of the light on these different days is well shown by comparing columns two, three, four, and five of the table.

The meteorological importance of such observations as these is suggested by the fact that it is commonly believed that a deep blue sky, reflecting little light, indicates the presence of a large amount of the vapor of water in the atmosphere, and the probable approach of rain, and that a very clear night frequently precedes a rainy day. If

it can be proved that such a relation exists between the relative moisture of the upper atmosphere and the light of the sky, it is evident that we have here a hygrometer with a widely extended range.

Sun's Dist.	I.	II.	III.	IV.	1.	2.	3.	4.	Mean.
0°30′		28.70	12.61	9.41		— 3.8	+ 5.5	—19.0	— 0.1
0 45	75.80	17.41	6.75	7.28	—15.2	+ 3.0	— 5.2	+ 6.4	+ 1.4
1		13.71	5.15	4.61		+22.0	+ 6.7	— 5.2	+ 7.8
1 15	61.16	7.88	3.83	3.53	— 4.8	— 4.3	+ 4.7	— 6.4	— 2.0
1 30		5.63		3.00		—12.5		— 0.0	— 6.2
1 45	47.33	4.70	2.53	2.37	— 1.8	— 8.2	+ 9.0	— 4.5	— 1.2
2		4.28	1.87	1.78		+ 1.9	— 4.0	—25.3	— 9 1
2 15	40.00	3.45	1.58	1.51	— 3.3	— 3.3	— 5.5	—21.7	— 8.4
2 30		2.87		1.37		— 6 4		—16.5	—11.4
2 45	34.26	2.41	1.15	1.23	+11.2	—10.2	—10.4	—15.0	— 6.1
3 15	27.16	2.05	1.01	1.07	+10.5	— 1.7	— 1.9	— 9.6	— 0.6
3 45	20.00	1.61	.83	.98	— 1.0	— 5.7	— 2.1	+ 0.0	— 2.2
4 15	17.66	1.50	.75	.88	— 2.4	+ 6.0	+ 4.3	+ 4.7	+ 3.1
4 45	14.75	1.33	.65	.83	+ 1.0	+ 9.7	+ 4.3	+12.3	+ 6.8
5 15	13.50	1.17	.60		+ 6.4	+11.5	+ 9.7		+ 9.2
5 45	11.33	1.10	.55	.72	+ 1.9	+18.6	+13.3	+23.1	+14.2
6 15	9.50				— 4.7				— 4.7
6 45	8 48			.60	— 5.2			+24.5	— 1.4
7 15	6.80			.56	—19.4			+26.8	+ 3 7
7 45	6.58			.52	—12.3			+29.0	+ 8.4
8 15	6.21				— 9.0				— 9.0
8 45	5.45				—13.8				—13.8
9 15	5.13				—12.3				—12.3
9 45	4.93				— 8.0				— 8.0
10 15	4.70				— 6.2				— 6.2
10 45	4.93				+ 5.7				+ 5.7
11 15	4.60				+ 4.7				+ 4.7
11 45	4.30				+ 1.6				+ 1.6
12 15	4.15				+ 6.4				+ 6.4
12 45	3 90				+ 6.5				+ 0.5
13 15	3.71				+ 6.7				+ 6.7
13 45	3.47				+ 4.5				+ 4.5
14 15	3.10				— 1.4				— 1.4
14 45	2.90				— 4.0				— 4.0
15 15	2.80				— 3.0				— 3.0
15 45	2.71				— 1.4				— 1.4

A curve has been constructed for each series of observations, having intensities as ordinates and natural sines of the sun's angular distances as abscissas; and an inspection of these curves shows a close agreement in their forms, which indicates that, notwithstanding the great differences in the intensity of the light, its variations followed the same, or nearly the same law, in each case. Other curves were constructed, with co-ordinates equal to the logarithms of the co-ordinates of the curves just mentioned, which show by their approximation to straight lines that the law of the variation of the light may be expressed by

the equation $y = mx^n$, the light being proportional to some power of the sun's angular distance.

The most noticeable deviation from a straight line is in the curve for series I. where it approaches the axis of y; here the intensities are less than required by a straight line, which is explained by the fact that this series was made late in the afternoon of a winter day when the light of the sun itself was rapidly diminishing, and, as before stated, the observations nearest the sun were made last.

Neglecting, for the reason just given, the first three observations of series I., we obtain the following as the most probable values of n in each case: series I. — 1.4; II. — 1.4; III. — 1.32; IV. — 1.21. Computing now the numerical value of m for each series, and substituting in the equation $y = mx^n$, we have for series: I. $y = 359x^{-1.4}$; II. $y = 33.1x^{-1.4}$; III. $y = 13.7x^{-1.32}$; IV. $y = 12.6x^{-1.21}$.

Columns 1, 2, 3, and 4 give the deviations from the formulæ of the observations of series I., II., III., and IV. respectively; the deviations being expressed in percentages of the intensities. The last column gives the mean of the deviations; neglecting, as before, the first three observations of series I. Although some of the deviations are quite large, yet the sums of the positive and negative deviations are approximately equal; and it will be observed that they frequently change their sign, which shows a close agreement with theory. It is probable that the larger deviations are attributable in part, at least, to sudden changes in the reflecting power of the sky, such as would be produced by air currents or by the precipitation or dissipation of atmospheric moisture.

That these deviations are greater than those due to errors of observation, is clearly shown by the experiments in the preceding article.

V. — LIGHT ABSORBED BY THE ATMOSPHERE OF THE SUN.

By E. C. Pickering and D. P. Strange.

The following series of experiments were made for the purpose of determining the relative amount of light received from portions of the sun's surface at varying distances from the centre of its disk. For this purpose, the sun's rays were reflected into a darkened room by means of the black glass mirror of a porte-lumière, and an image of the sun, 40 cms. in diameter, was, by means of a small telescope, thrown upon a screen placed at a distance of 230 cms. from the aperture. In the

centre of this screen was cut a circular hole 2 cms. in diameter, and the light passing through this aperture was received upon a Bunsen's photometer disk, placed at a distance of 25 cms. behind it. The porte-lumière was then moved until the desired portion of the image coincided with the centre of the aperture in the screen, and the image kept at rest by a slight movement of the telescope whilst the photometer reading was taken. The light used for comparison was a standard candle, which was placed in the photometer described above (page 421). Much difficulty was experienced on account of the difference in color of the light from the sun and candle, in obtaining a satisfactory disappearance of the spot of the photometer disk. Various attempts were made to remedy this trouble, by using colored paper, disks of colored plaster, &c., none of which succeeded very well, and the ordinary white disk was finally adopted. A large number of preliminary series of observations were made, and rejected as not being sufficiently accurate.

The results of the last three days' observations are given in Table I.; the first column giving the percentage distance from the centre

TABLE I.

	1st D.	2d D.	3d D.	E.	P.	M.	P. Er.	Theor.	Diff.
0.0	100.0	100.0	100.0	100.0	100.0	100.0	0 0	100.0	0.0
12.5	98.5	98.9	99.1	99 0	98.9	98.9	0.1	99.4	—0.5
25.	94.3	97.5	94.4	97.2	97.0	96.7	0.4	97.9	—1.2
37.5	93.9	94.7	91.0	93.7	93.1	94.2	0.3	95.0	—0.8
50.	90.9	91.7	88.6	90.8	89.5	91.3	0.3	91.0	+0 3
62.5	84.4	86.9	79.7	87.3	83.8	86.2	0.7	85.0	+1.2
75.	80.5	78.9	76.0	80.5	76.0	78.8	0.7	77.2	+1.6
85.	69.1	69.5	66.5	71.4	66.3	69.2	0.8	68.4	+0.8
95.	62.9	53.6	57.9	56.1	52.8	55 4	1.6	54.8	+0.6
100.	....	....	87.4	....	....	37.4	0.9	37.4	0.0

towards the edge, and the succeeding ones the intensity of the light compared with that of the centre taken as unity. The second column gives the mean of the first day's, the third of the second day's, and the fourth of the third day's observations. Upon the second day, there were several times as many observations taken as upon either of the others, and its mean is correspondingly more reliable. A portion of the observations were taken upon or near the polar, and others upon or near the equatorial, diameter of the sun. Column 5 gives the mean of the measurements taken upon the equatorial, and number 6 the mean upon the polar, diameter. As these are the results of but a

comparatively small number of observations, it seems difficult to decide whether the apparently greater brilliancy towards the edge along the equatorial diameter is real, or is due to errors of observation. Column 7 gives the mean of all the measurements taken, and column 8 the probable error of this mean.

If the sun had no atmosphere, its disk as seen from a distance would appear uniformly bright, since the light emitted by one square metre in any given direction is inversely as the cosine of the angle of emission, while, owing to foreshortening, its apparent area is proportional to the cosine of the same angle. Let us next suppose it surrounded by a homogeneous atmosphere not perfectly transparent. Evidently the absorption will depend on the distance which the light has to pass through it, and will be greatest at the edges, and least at the centre; or the disk will appear brightest at the centre and darkest at the exterior, as is actually the case. To determine the law of this variation, let the radius of the disk equal unity, x the apparent distance of any point from the centre, h the height of the atmosphere, b the brightness of any portion of the disk were there no atmosphere, a the proportion of light which would traverse a thickness of the atmosphere equal to unity, or to the sun's radius. Call v also the distance the light from the point x must traverse before emerging from the solar atmosphere, and y the apparent brightness of the same point. It is readily proved that $v = \sqrt{(1+h)^2 - x^2} - \sqrt{1 - x^2}$, and that $y = ba^v$; therefore,

$$y = ba^{\sqrt{(1+h)^2 - x^2} - \sqrt{1-x^2}}$$

is the equation which gives the brightness of any point of the sun's disk, assuming that its atmosphere is homogeneous. From any three corresponding values of x and y we can compute a, b, and h. Assuming from the above observations $y = 1$ for $x = 0$, $y = .782$ for $x = .75$, and $y = .374$ for $x = 1$, and taking logarithms, we deduce the three equations of conditions: —

$$0 = \log b + h \log a;$$
$$-.1068 = \log b + (\sqrt{(1+h)^2 - .5625} - .6614) \log a;$$
$$-.4271 = \log b + (\sqrt{2h + h^2}) \log a.$$

Subtracting these equations, we eliminate b; and dividing one of the resultant equations by the other, eliminates a. We thus deduce the equation: —

$$\sqrt{.4375 + 2h + h^2} - .25\sqrt{2h + h^2} - .75h - .6614 = 0.$$

To solve this equation, its first member was placed equal to m, and various values of h substituted; a curve was then constructed with m and h as coördinates, and a few trials readily gave the value of h corresponding to $m = 0$. This affords an easy means of solving many equations not readily treated by the usual methods. The value of h thus found was a little less than unity. Substituting $h = 1$ gives $\log a = -.5835$, $a = .2609$, $\log b = .5835$, and $b = 3.833$. Or, if the effect of the solar atmosphere resembles that of a homogeneous atmosphere, its height must equal the radius of the sun, and its opacity be such that the light in the centre is only .26 of what it would be were the atmosphere removed; or the sun's brightness in the latter case would be throughout 3.8 times its present brightness at the centre Substituting these values of a, b, and h in our first equation, gives

$$\log y = .5835 - .5835\,(\sqrt{4 - x^2} - \sqrt{1 - x^2}),$$

in which, by substituting various values of x, we deduce the corresponding values of y, the light at various points of the sun's disk. In Table I., the column headed Theor. gives the amount of light computed by this formula, and the last column the differences from the mean observations, M. Three other theoretical values were computed for these points, but those given in the table were retained as agreeing most nearly with observation. From these it appeared that a considerable variation in h did not alter the amount of light very materially, that a diminutive change of h of one-tenth increased the light between $x = .6$ and $x = .9$ only half a per cent, and for other values of x altered y still less. Moreover, the differences in the last column of the table are evidently too regular to be due to accidental error, but rather show a real variation from theory, due to the fact that the atmosphere is not really homogeneous. We might assume that the law of the density is the same as that of the earth's atmosphere, or that, the height being taken in arithmetical progression, the densities will vary geometrically. But this leads to an equation which cannot be integrated, and, moreover, cannot be correct in fact, since it assumes that the temperature is uniform throughout. The great heat near the surface, by expanding the atmosphere in contact with it, diminishes its density, thus rendering it more nearly homogeneous than the above law would require; this effect is, however, counteracted by the tendency of the heavier gases to descend.

It is a matter of interest to know not merely how much light is cut off by the atmosphere at the centre of the sun's disk, but also how much the whole light of the sun will be reduced by the same cause.

Suppose the curve constructed with coördinates equal to x and y of the preceding table, and that a solid of revolution is generated by revolving it around the axis of Y: evidently, the volume of this solid will represent the total amount of light received by the observer from the whole of the sun's disk, and the volume of the circumscribing cylinder will equal that which would be received if the disk throughout had the same brightness as at the centre. The ratio of these two quantities is, however, obtainable by Simpson's formula, and gives the result 82.6, or the light is about five-sixths of what it would be if the disk had the same brightness at the edges as at the centre. Now, as shown above, the light at the centre is reduced by the atmosphere to 26.1 per cent. Hence the total reduction of the whole surface is $.261 \times .826 = .216$. And, since the light is reduced in every direction by the same amount, we may say that the sun would give out 4.64 times as much light if its atmosphere were removed.

The results of this paper may therefore be summed up as follows. The light of the various parts of the sun's disk is measured by the modification of the Bunsen photometer here employed, and given in the accompanying table, with a probable error not exceeding one per cent except close to the edge. The light at the edge is about .4 of that at the centre. The variations in brightness are nearly those which would be produced by a homogeneous atmosphere of height equal to the sun's radius, and opacity such that only 26 per cent of the light is transmitted. There appears to be a slightly different distribution of the light along the polar, from that along the equatorial, diameter. If the atmosphere were removed, the brightness of the sun's disk would be uniform, and 3.83 times that of the centre of the disk at present. Moreover, the total amount of light would be increased 4.64 times.

VI. — TESTS OF A MAGNETO-ELECTRIC MACHINE.

By E. C. Pickering and D. P. Strange.

The rapidly increasing use of magneto-electric machines as a source of electricity renders accurate tests of the comparative advantages of the various forms and exact measurements of the currents generated under varying conditions very desirable. The machine employed in the following experiments was made by Mr. M. G. Farmer, and consists of a large electro-magnet wound with four coils soldered together

at the ends, like four battery cells connected for quantity. Between the poles of this magnet a Siemens' armature is revolved, and both magnet and armature are included in the main circuit. The instrument is therefore extremely simple, and, when the circuit is broken, requires no power to run it except to overcome the friction of the bearings. The total weight is about 700 lbs., and the dimensions 33.5 by 21.5 inches (85 $\times$ 55 cms.), with a height of 14 inches (37 cms.). To avoid heating, a water space is left close to the armature, but this is required only when the resistance of the circuit is small.

The quantities to be measured were as follows: 1st, velocity of rotation of armature; 2d, power required; 3d, strength of current with various speeds and resistances; 4th, electro-motive force under the same conditions; 5th, when the current is used to produce a light, a measure of the latter is candle-power.

Power. The boilers and engine of the Institute were used as a source of power. The nominal capacity of the boilers was sixty-six, and of the engine fifteen horse-power; but, owing to various difficulties beyond the control of the writers, only a small portion of this was available, and that only for limited periods of time. A belt passed from the fly-wheel of the engine over a countershaft in the Physical Laboratory, giving it a velocity of about 500 turns per minute. A set of five cone pulleys were attached, by which a speed of 333, 410, 500, 610, and 750 turns could, by shifting a belt, be given to a second shaft. The latter carried a wheel 20 inches in diameter, and drove the machine by a belt passing over a pulley 8 inches in diameter attached to the armature. As the speed of the engine varied somewhat, a speed of from 800 to 2100 turns per minute was thus obtained. Various plans were tried to measure the power employed. For the earlier experiments a Batchelder dynamometer was used, in which the motion was transmitted through four bevel-gears, and the moment of tension measured by a spring-balance and weights. The instrument was not however intended to be run at such high speeds, and the gears were very noisy.

Speed. The number of revolutions per minute is so important a factor in these measurements that it must be constantly determined. At first, a common shaft speeder was employed; but, apart from its want of accuracy, its constant use was laborious, and it showed only the total number of turns during a minute, and not the speed at any intermediate instant. A device was accordingly employed, constructed by Mr. J. B. Henck, Jr., by which these difficulties are completely avoided. The plan is not new, having been published in a modified

form many years ago in Nicholson's Mechanics and elsewhere. Three vertical gas-pipes are placed side by side, and connected together below; then half filled with mercury, and so mounted that they may revolve around the axis of the central pipe; a glass tube filled with water is attached to the latter, and serves to show the position of the mercury. Motion was transmitted to the whole from the horizontal shaft of the machine by a spiral spring, as in a dental lathe, but afterwards this was replaced by a pair of bevel-gears. If now the machine is set in motion, the mercury is by centrifugal force thrown from the central to the outer tubes, and the water in the glass tube falls. A graduated scale shows the position of the water, which remains very constant as long as the velocity is uniform, and by its motion shows the slightest variation in speed. The reduction is effected by noting the water level with various velocities as measured by a shaft-speeder, and constructing a curve with coördinates equal to these two quantities. If the tubes are exactly parallel and of uniform diameter, this curve will be a parabola, with axis vertical and parameter determined by the equation $y = 473 \times 10^{-8} n^2 d^2$, in which d is the distance of the outer tubes from the centre in inches, and n the number of turns per second. Evidently an inch would correspond to a much greater change in velocity at high than at low speeds, and accordingly the open ends of the outer tubes were bent in towards the centre. This had the additional advantage of preventing the mercury from being thrown out, and of greatly increasing the range of the instrument. As actually constructed, the speed in turns per minute very nearly equalled the square of the depression of the water level in tenths of an inch.

Resistances. A difficulty at once presented itself in varying the resistance of the circuit, since resistance coils of the ordinary form would be at once injured or even melted by the immense quantity of electricity transmitted. Accordingly a set of resistances were prepared by stretching some uncovered German silver wire along the wall of the laboratory, so as to form nine loops, of eighty feet each, of No. 28 wire. As the diameter is .017 inches, the surface exposed to radiation is about 460 square inches; and, as the air circulates freely around them, there is no difficulty from their heating, even when the machine is connected directly with their terminals. Each of these loops has a resistance of 36.9 ohms, and one or more may be thrown into circuit by a switch. For smaller resistances, a similar device was employed. A frame, 3 feet wide and 6 feet high, was covered on both sides with horizontal wires, passing around screws so as to form 30 loops of No. 22 wire (diameter .029″) and 55 loops of No. 16 wire

(diameter .065″). The former had a total resistance of 29.68 ohms, the latter of 9.18 ohms, or the single loops .99 and .167 ohms. To allow for accidental variations in the wire or in the connection, each loop was measured separately, and a table of resistances thus formed. To the end of each loop was attached a short piece of stout copper wire, wound in a helix and sunk in a hole, so that any two could be connected by a wire terminating in copper plugs. By this system, any resistance from .17 ohms to 370 ohms was easily thrown into the circuit, and the connections were few in number and of very small resistance. This plan also had the advantage of extreme cheapness. The heating of the wire was independent of its length, except so far as the current altered. Practically, the three sizes of wire employed would convey 1.5, 5, and 10 vebers, without undue heating. A change of temperature of 100° C. increases the resistance of German silver wire about 4 per cent; and, to allow for this, a so-called thermometer-board was employed, on which pieces of the three wires wound in a helix were stretched. To determine the heating of either size of wire, the proper helix was inserted in the circuit, and a thermometer placed in it. On trial, it was found that the readings were much too high, the radiation prevented by the adjacent spires of the helix much more than compensating for the imperfect connection with the thermometer. This difficulty might be avoided by stretching the helix until these two errors should compensate, which might be tested by covering the helix and a straight wire with iodide of mercury and copper, and altering the form until the color of the iodide changed with the same current in both. As, however, the correction is small at ordinary temperatures, it was deemed best to neglect it, taking care to touch them occasionally when very powerful currents were passing, to make sure that the wires did not become very hot.

Current. A special device is also needed for the determination of the current produced in absolute measure. If an ordinary tangent galvanometer with a single coil of thick wire was employed, the stronger currents could be well compared; but it would be difficult to reduce them to vebers, since a feeble current suitable for depositing copper would not appreciably deflect the needle. Accordingly, a cosine galvanometer shunted was employed, or rather, as here used, a tangent galvanometer, since the coils were kept vertical. The coils consisted of about 50 turns of No. 16 copper wire, 6 inches in diameter, and 3 inches apart. The needle had a length of but $\frac{3}{8}$′, and was made of a piece of watch-spring. An index, 3 inches long, was attached; and the magnet, being suspended by a filament of silk, swung over a gradu-

ated circle divided into degrees, and the fractions estimated to tenths by the eye. To eliminate parallax, the bottom of the compass-box was formed of looking-glass, and the eye so placed, when the reading was taken, that the index and its reflection coincided. To determine the constant of the galvanometer, a constant current from a thermal battery was passed through it and through a beaker containing sulphate of copper, and the weight of copper deposited measured. Two determinations were made, and gave the result .052. To make sure that the galvanometer followed the law of the tangents, a series of resistances were interposed in the circuit, and the deflection measured. The results showed that the error was extremely small, even for angles as large as 80° to 85°. The resistance of the galvanometer was .22 ohms, and by it, currents from .02 to .3 vebers could be well measured. For stronger currents a set of shunts were prepared. The wires from the galvanometer were carried parallel to each other and near together for some distance to avoid their disturbing action on the needle, and the resistance thus increased to exactly .25 ohms. Three shunts, A, B, and C, were then prepared, which should reduce the current to .2, .04, and .014, consisting of short stout pieces of German silver wire. The first and second of these were easily made by computing their required resistance, and sliding them in or out of the screw cups in which they were held. They were then tested by passing the same current first through the galvanometer with and without the shunt, and comparing the tangents of the deflections in the two cases. To correct for the change in resistance, an additional resistance was inserted when the galvanometer was shunted. The third shunt could not be made directly, as its resistance was only .0034 ohms, and we could measure directly, only to thousandths of an ohm. The method of comparison alone was therefore used, reading the deflection when the whole current of the machine was passing, and again using the $\frac{1}{25}$ shunt. The correct values of the three shunts were thus found to be .1980, .0392, and .0137. The latter consisted of a bar of German silver, .13″ in diameter and 3″ long. To pass from one shunt to another, a simple switch or plug could not be employed, since the resistance of the shunts B and C was so small that the variable resistance thus introduced would become quite perceptible, being multiplied many times; and, moreover, with the stronger currents, the points of contact might become heated or burnt. Accordingly, a switch was inserted in the wire connected with one terminal of the galvanometer, by which it could be connected with either of the three shunts, and a second connection made with each, and with the main circuit. The

other terminals of the machine and galvanometer were permanently connected with the other ends of the three shunts.

Another and better method, both as requiring no very small resistances and as employing but a single switch connection, is the following. Call G the resistance of the galvanometer, connect a resistance r' to one of its terminals, and shunt by a second resistance s'. Attach to one end of this a coil r'', and shunt again by the coil s''. If necessary, shunt again until a sufficient reduction is attained. Now connect one terminal of the machine with one end of G, s', and s'', and bring the other in contact with the other end of either of them by a simple switch, and we shall have the effect of three shunts of three different sensibilities. The total resistance and the relative constants may be computed in each case, or they may be measured directly. Calling the total resistances R_1, R_2, and R_3, and the shunts to which they are equivalent S_1, S_2, S_3, we may deduce proper values by the usual formulas for divided currents. As, however, the case is a little complex, it is best to reduce it to the following symmetrical form: Let $f(x, y, z) = xy + xz + yz$; then we have:—

$$R_1 = G\frac{f(r', s', s''+r'')}{f(G+r', s', s''+r'')} \qquad S_1 = \frac{f(r', s', s''+r'')}{f(G+r', s', s''+r'')}$$

$$R_2 = \frac{(G+r')\,s'\,(s''+r'')}{f(G+r', s', s''+r'')} \qquad S_2 = \frac{s'\,(s''+r'')}{f(G+r', s', s''+r'')}$$

$$R_3 = S\frac{f(G+r', s', r'')}{f(G+r', s', s''+r'')} \qquad S_3 = \frac{s'\,s''}{f(G+r', s', s''+r'')}.$$

In these equations, G would generally be given; and we may, theoretically at least, assume any five other quantities, and then deduce the remainder. As, however, these equations are too complex to be used with any convenience, let us see how they may be simplified. Suppose that $s' = s'' = G$, and that $r' = r'' = nG$, then our six equations become:—

$$R_1 = G\frac{1+3n+n^2}{3+4n+n^2} \qquad S_1 = \frac{1+3n+n^2}{3+4n+n^2}$$

$$R_2 = G\frac{1+2n+n^2}{3+4n+n^2} \qquad S_2 = \frac{1+n}{3+4n+n^2}$$

$$R_3 = G\frac{1+3n+n^2}{3+4n+n^2} \qquad S_3 = \frac{1}{1+3n+n^2}.$$

If now $n = 1$, or all the resistances equal G, the three values of R become .625, .5, and .625; while those of S are .625, .25, and .2. If $n = 2$, R becomes .733 G, .6 G, and, .733 G, S_1 .733, .2, and .091; $n = 5$

gives R_1 .853 G, .75 G, and .853 G, and S .853, .125, and .0244. Finally $n = 10$ gives R_1 .916 G, .846 G, and .916 G; and C_1 .916, .076, .0073. It will be more convenient in general to give r', r'', s', s'', approximately the proper values, and then measure R_1, R_2, and R_3 by the Wheatstone's bridge. Next interpose resistances in the wires attached to the switch, so as to make the total resistance of the galvanometer the same for all positions of the switch. Thus, in the above example, when $n = 10$, if resistances of .084, .154, and .084 ohms are interposed, the galvanometer resistance will be an ohm in each case. The values of S_1, S_2, and S_3 are now found directly by comparing the deflections when the switch is moved. By this device, the range of a tangent galvanometer may be increased indefinitely, and the strongest as well as weak currents measured by it. Moreover, the resistance is not altered, so that readings with different shunts are directly comparable.

The first experiments made with the machine were for the purpose of determining whether the current was constant under the same circumstances or not. It was feared that, as the magnetism was induced by the current itself, variations would appear, dependent on the time during which the circuit had been closed; but, on trial, it was found that the magnet attained its full polarity sooner than the needle of the galvanometer came to rest, and that, on making and breaking the circuit, the successive deflections were almost precisely equal. The next problem was to determine the effect upon the current of changing the position of the commutator. This is so made as to be capable of being revolved round the shaft of the Siemens' armature through an angle of about 45°, thus taking off the current when the coil of the armature is in different positions relatively to the electro-magnet. Observations were taken with the commutator in the following eight different positions: No. 1 is with the commutator turned farthest down, or with its plane as nearly parallel with the plane of the electro-magnets as possible. It is then turned up through an angle of about 6.5° with each succeeding number. In No. 8, it is very nearly perpendicular to the plane of the magnets. The results of several experiments are given in Table I., in the 2d, 3d, 4th, and 5th columns, of which the current obtained in the various positions is given in vebers per second. In the last four columns, the currents are given in percentages of the maximum obtained.

TABLE I.

1.	2.	3.	4.	5.	6.	7.	8.	9.
1	.0620	.0962	.0436	.1066	96.6	93.8	93.2	92.5
2	.0625	.0986			97 3	96.1		
3	.0631	.0999	.0450	.1115	98.3	97.4	96.2	96.8
4	.0638	.1010	.0455	.1115	99.4	98.3	97.2	96.8
5	.0641	.1021	.0467	.1146	99.8	99.7	99.8	99.5
6	.0642		.0468	.1152	100.0		100.0	100.0
7	.0640				99.7			
8	.0635		.0466	.1152	98.9		99.6	100.0

It thus appears that the position of the commutator has but little influence upon the strength of the current; but, as the maximum was obtained in each case from position No. 6, it was kept in that place in all further experiments.

Next, to determine the relation between the four variables, speed of revolution, resistance in circuit, current, and electro-motive force. An attempt was also made to measure the work required to run the machine, and the coefficient of efficiency; but, from lack of proper dynamometric facilities, the attempt was necessarily abandoned after the first series of experiments.

The results of these experiments are given in the following tables, in which R is the resistance of the circuit, expressed in ohms; S is the speed, or number of revolutions of the armature per minute; C is the current in vebers per second; E is the electro-motive force in volts; E_1 is the computed electro-motive force in volts, which would have been obtained with a speed of 1,000 revolutions per minute; W is the work expended, in foot-pounds, including friction. $W.C$ is the work the current is capable of doing, in foot-pounds; and $C.E$ is the coefficient of efficiency of the machine, obtained by subtracting the work required to drive the machine on an open circuit from the actual work W, and dividing the computed work $W.C$ by the remainder.

From an examination of these tables, several important conclusions may be drawn. For large resistances, over 38 ohms for instance, the electro-motive force is nearly proportional to the speed, and is given by the equation $E = .007S$. The advantage of placing the magnet in the main circuit is here in a great measure lost, since the large outside resistance so far reduces the current that its effect on the magnet is slight. The constant .007 affords a good means of comparing various machines of this form, since its magnitude depends directly on the arrangement of the magnet and armature. For resistances less than

TABLE II.

R.	S.	C.	E.	E1.	W. C.	W.	C. E.
264.6	750	.023	6.10	8.13	.104	806	
227.4		.027	6.09	8.11	.121	803	
190.3		.032	6.08	8.11	.143		
153.2		.040	6.08	8.11	.178		
116.0		.052	6.10	8.13	.238		
78.9		.078	6.16	8.22	.335		
52.6		.131	6.88	9.04	.664		
41.7	760	.159	6.67	8.76	.788		
41.5		.162	6.72	9.02	.816	809	
27.6	745	.336	9.28	12.45	2.34	828	
21.8	745	.587	12.79	16.9	5.64		
15.44	740	1.42	21.14	28.5	22.5	550	.096
11.44	730	2.20	25.16	34.5	41.5	697	.105
7.50	725	4.17	31.29	43.1	97.9	955	.146
5.15	725	5.29	27.24	37.5	108.1		
4.44	720	6.32	27.48	38.1	124.9	1274	.122
4.44	580	6.96	30.91	53.3	161.4	984	.228
2.97	425	11.26	38.39	78.5	282.1	952	.362
2.78	590	8.62	28.96	40.6	154.9	1403	.133
2.30		9.71	22.32	39.8	162.4		
1.956	530	10.05	19.66	37.1	148.2		
1.786	310	11.58	20.68	66.7	179.6	785	.284
1.656	520	11.30	18.68	35.9	158.3		

TABLE III.

R.	S.	C.	E.	E1.
333.6		.020	6.62	6.96
296.5	950	.022	6.58	6.91
259.3		.026	6.64	7.00
222.5		.030	6.62	7.02
185.7	940	.036	6.67	7.09
148.8		.045	6.73	7.13
111.6	945	.061	6.85	7.25
74.5		.092	6.87	7.29
37.95	940	.206	7.80	8.29
30.48	925	.476	14.51	15.7
26.57	925	.594	15.78	17.0
21.65		.899	19.47	21.1
16.70	920	1.96	32.73	35.6
13.76		2.72	37.42	41.1
11.77	905	3.41	40.11	44.3
9.81	900	4.39	43.07	47.9
8.94	890	5.06	45.25	50.9
7.80	870	5.86	45.70	52.6
6.92	840	4.90	33.9	40.4
5.91	670	6.17	22.3	33.3
3.93	550	6.76	26.6	48.4
2.95	510	9.40	27.7	54.4

TABLE IV.

R.	S.	C.	E.	E_1
333.6	1170	.025	8.31	7.11
296 5		.028	8.31	7.11
259.3		.032	8.33	7.14
222.5		.037	8 34	7.19
185.7	1160	.045	8.36	7.21
148.8	1140	.057	8.41	7.38
111.6	1120	.076	8.44	7.54
74.5	1110	.115	8.54	7.70
37.95	1100	.230	8.73	7.98
74.5	1140	.127	9.44	8.29
37.95	1140	.252	9.58	8.61

TABLE V.

R.	S.	C.	E.	E_1.
333.6	1330	.030	9.93	7.47
296.5		.033	9.86	7.42
259.3	1320	.038	9.80	7.42
222.5	1320	.044	9.88	7.49
185.7	1325	.053	9.86	7.46
148.8	1325	.066	9.83	7.45
111.6	1320	.090	10.05	7.62
74.5		.137	10.20	7.73
37.95	1325	.293	11.14	8.56
21.2	1380	1.37	29.01	20.86
16.3	1365	1.99	32.26	23.2
12.3	1350	3.15	38.7	28.7
8.6	1300	4.98	42.8	32.9
7.6	1280	5.78	43.9	34.3
6.6	1230	6.96	46.0	37.4
5.6	960	7.17	40.2	41.8
4.8	880	7.75	36.5	41.4
4.1	850	8.11	33.2	39.1

TABLE VI.

R.	S.	C.	E.	E_1.
333.6	1620	.036	12.01	7.41
296.5	1615	.040	11.92	7.38
259.3		.047	12.27	7.56
222.5		.054	12.02	7.42
185.7	1630	.065	12.05	7.39
148.8		.082	12.19	7.48
111.6	1625	.109	12.22	7.57
74.5		.165	12.29	7.61
37.95	1625	.332	12.60	7.80
26.2	1675	1.31	34.31	20.5
21.2	1675	1.82	38.59	23 1
16.3	1675	2.45	40.01	24.0
12.3	1635	3 42	47.06	28.8
8.6	1260	4.82	41.47	54.4

TABLE VII.

R.	S.	C.	E.	E_1.
333.6	2010	.041	13.7	6.81
296.5		.046	13.6	6.78
259.3		.053	13.7	6.93
222.5		.063	13.9	6.81
185.7	2010	.074	13.7	6.77
148 8		.092	13.6	6 62
111.6	2015	.120	13.4	6.65
74.5		.174	13.0	6.39
37.9	2015	.317	12.0	5.96

38 ohms, the electro-motive force rapidly increases by an amount which is approximately given by the formula, $E = S\,(.042 - .0009\,R)$, from which we see that the electro-motive force continually increases as we diminish the resistance, and, if the resistance could be reduced to zero, would attain the value $E = .042\,S$.

The column E_1 is computed by assuming the electro-motive force proportional to the velocity. This column can be used more conveniently than that marked E, since with small resistances the power required was so great as to make the belts slip, and greatly diminish the speed.

In Table II. some measurements of the power are given, as also the ratio of the theoretical power to that actually employed. The latter was measured by the dynamometer, the former computed by the very convenient theoretical formula, $W = \frac{3}{4}\,CE$. From the results, it will be seen that, for large resistances, the power employed, beyond that required to drive the machine, is insignificant, but rapidly increases as the resistance diminishes; the efficiency also at the same time increasing and attaining its greatest value with the smallest resistances. Of course, the absolute efficiency, or ratio of electricity generated to power expended, would be still less than this, being very small for large resistances, and attaining a maximum of about 30 per cent. When we consider, however, how large an amount of work is consumed by even a small amount of heat, the coefficient in the above cases must be regarded as large.

A series of experiments was next made to determine the strength of the current generated in different positions of the armature. The apparatus was constructed by Mr. S. J. Mixter, and consisted of a wooden wheel attached to the armature, and revolving with it. On this rested a brass wire; and a strip of copper was inserted in the wheel, so that it established contact between the axle and the wire, through an angle of about 10°. The latter was supported by a second larger

wooden wheel, which could be turned and held in any desired position by inserting a pin in one of a series of holes in its circumference, at intervals of 10°. The experiment was performed by connecting the brass wire and axis of the machine with the galvanometer, so that during each revolution of the armature the current would be for an instant diverted through the galvanometer, these currents following each other so rapidly when the machine was running as to produce a sensibly constant deflection. The larger wheel was then turned 10°, and the observation repeated. The 0° and 180° of this wheel correspond to the points where the circuit is reversed by the commutator.

TABLE VIII.

P.	C.	C.
0	.0498	.1015
10	.0508	.0912
20	.0378	.0786
30	.0338	.0693
40	.0284	.0620
50	.0257	.0514
60	.0211	.0399
70	.0159	.0392
80	.0136	.0385
90	.0130	.0392
100	.0141	.0446
110	.0152	.0588
120	.0188	.1000
130	.0348	.1329
140	.0638	.1406
150	.0715	.1367
160	.0678	.1260
170	.0629	.1162
180	.0498	.1015

Table VIII. gives the result of two series of experiments of this kind, the wheel being turned through 360° and the mean of the two readings at intervals of 180° taken. Column 1 gives the angle through which the wheel has been moved, and column 2 the current, the main circuit having a resistance of 16.7 ohms, and the galvanometer circuit a resistance of 1.3 ohms. Column 3 in like manner gives the current when the resistance of the main circuit is reduced to 10 ohms. An examination of this table shows that the current at no point becomes zero, but varies from a maximum at about 145° to a minimum at 90°. If the distance of the poles of the magnet was large compared with the motion of the armature, the current would vary as the sine of the angle, supposing that there was no induction or

other disturbing cause. Accordingly, the current would become zero at two points midway between its two maxima, and this would be the point where the commutator should be placed. In that case, no spark would be seen at the commutator, since the circuit would be broken only when the current was zero. In practice, it was found that there was no portion of the commutator where the spark could be entirely avoided when the resistance was small, evidently owing to the fact shown by these observations, that the current at no point is zero. Moreover, on constructing the curves with coördinates equal to the angles and currents, it will be seen that the inclination is much greater before than after the maximum ; so that the latter, as stated above, is distant only about 55° from the minimum, instead of 90°. The cause of the deviation from the curves of sines is probably the current induced by the magnet, which adds or subtracts its effect according as the current is increasing or diminishing.

In trying experiments upon the light produced by the current, several difficulties were encountered. One of the most serious of these was from the slipping of the driving belts, when the machine was running at high rates of speed and the circuit was made through so small a resistance as the regulator and light. From this cause, we were unable to obtain a steady speed of more than, 1,300 revolutions per minute, which was not sufficient to give the best results. A further difficulty was experienced from the great difference in power required to run the machine when the current was passing, and when the carbons became so far separated that the current was unable to pass. A change of probably 4 or 5 horse-power was thus almost instantly made, whenever the current was made or broken, and the consequent shock upon the machinery was very great. It also appeared that the form of regulator used (Duboscq's) was not capable of controlling the current so that the light should be steady. When the carbons were brought in contact, the current was so great that the magnet acted strongly, starting the reversing clock-work and separating them half an inch or more. This broke the circuit, and the machine began to revolve very rapidly ; soon the carbons were brought together, throwing a great strain on the engine, and thus they oscillated, producing a very bright light for an instant and then extinguishing it. Better results would probably be attained without the reversing arrangement, by a change in the magnet of the regulator, or by increasing the electromotive force of the current. Some results were however obtained by a very careful adjustment of the spring holding the armature. With a velocity of 1,130 revolutions, a tolerably constant light was obtained.

Current, 3.65 vebers. Resistance in circuit, about 10 ohms. Resistance of light, 3.3 ohms. With a speed of 1,325, total resistance 9 ohms, and current 5.71 vebers, a light varying from 600 to 900 candle-powers was obtained. With a speed of 1,280, resistance 7 ohms, and current 5.20 vebers, the light varied from 650 to 900 candle-powers. Doubtless a much greater light could be obtained with a different regulator and means of obtaining a high speed.

The effects of the current were very fine, and have been frequently described in connection with the Wilde, Gramme, and other machines. Thick wires were melted, heavy weights sustained in the air in the interior of large coils, and excellent diamagnetic effects shown. The induced current on breaking the circuit was very severe when taken through the body, and the spark very long and bright.

The advantages of this machine are its simplicity, compactness, and small weight, compared with other machines of equal power; and little or no trouble was experienced from heating with the currents here employed. In conclusion, we wish to express our hearty thanks to Mr. Farmer for lending us the machine, and hope that we may be enabled to continue these experiments with this and other machines next year, if we can secure an adequate motor and proper means of measuring power.

VII.—ANSWER TO M. JAMIN'S OBJECTIONS TO AMPÈRE'S THEORY.

By William W. Jacques.

It is the purpose of this paper to answer some objections which M. Jamin has made to Ampère's theory of magnetism.

In the Comptes Rendus for Jan. 12, 1874, M. Jamin published the results of some experiments, in which he obtained the laws of the distribution of magnetism in a soft iron bar which formed the core of two coils by measuring the force necessary to detach an armature when placed at different points along the bar. He gives the equations to the curves obtained by sending an electric current through one of the coils, through both coils in the same direction, and through both coils in opposite directions; and finds that the force necessary to detach the armature at any given point is less when the currents are parallel than when opposed; from which he draws essentially the following conclusions: —

1°. If we admit the theory of solenoids, the action of parallel currents ought to be added, and the amount of magnetic intensity to be increased. The reverse takes place.

2°. When the currents of the coils are sent in opposite directions, they ought to act inversely on the particular currents of the iron, and the results should diminish each other. On the contrary, they are added.

3°. The action of the bobbins should, in this case, be nothing at the middle point of the bar. It is not so. We cannot say that there is, at this point, a resultant pole, for it would manifest itself by a point of repulsion.

M. Jamin then states that these results seem to him to require a modification in the theory of solenoids.

Mr. D. Sears has (American Journal, July, 1874) measured the distribution of magnetism in an iron bar which formed the armature of the cores of two coils, by sliding a coil of fine wire, whose terminals were connected with a galvanometer, along this armature, and measuring the instantaneous current induced in this secondary coil when the armature was magnetized by sending a current through the primary coils. His results are opposed to those of Jamin. The case, however, is not exactly that of Jamin, and I have therefore, after repeating Mr. Sears's experiments with similar results, applied this method of measuring the distribution of magnetism, by means of a coil of fine wire, to Jamin's apparatus, as follows: —

I made a bar of soft iron, 50 cm. long, the core of two coils, as in Jamin's experiment, and so connected the coils with a battery that a current could be sent through a single coil, or through both coils in the same or in opposite directions. One of Farmer's thermo-batteries was used as a source of electricity, because of the very great constancy of its current. A small coil of fine wire, like that used by Mr. Sears, was arranged to slide along the bar, and its terminals were connected with a Thomson's galvanometer. When a current was sent through the primary coils, magnetism was induced in the bar, and this, in its turn, induced an instantaneous current in the coil of fine wire, and so caused a deflection of the galvanometer. Although the secondary coil was parallel to the primaries, I found, by substituting a glass rod for the iron bar, that the direct action of the inducing coils on the secondary coil was exceedingly small, excepting when these were brought very near together, which it was not necessary to do in this experiment.

The method used in these experiments is more delicate than Jamin's, as may be shown by constructing curves from the observations in Table I., or by the smallness of the differences in the last column of that table;

ind, since we know the positions of the poles of a magnet relatively to a surrounding coil, we may determine the kind of magnetism, or, in other words, the direction of the Ampèrian currents in either half of the bar, which Jamin's method fails to do.

The results of this experiment, which are directly opposed to those of Jamin, and, therefore, tend to confirm the theory of Ampère, are given in the following table: —

TABLE I.

$x=$	Currents Parallel.	Currents Opposed.	Calculated Mean.	Single Bobbin.	Differences.
15	55	46	50.5	50.0	+ .5
18	33	24	28.5	30.0	—1.5
20	26	16	21.0	21.0	.0
21	23	12	17.5		
22	21	9	15.0		
23	20	6	13.0	13.0	.0
24	19	3	11.0		
25	19	0	9.5	8.5	+1.0
26	19	— 3	8.0		
27	20	— 6	7.0	6.5	+ .5
28	21	— 9	6.0		
29	23	—12	5.5		
30	26	—16	5.0	4.5	+ .5
32	33	—24	6.5		
35	55	—45	5.0	3.0	+2.0

The first column gives the distances from the left end of the bar; column two gives the deflections of the galvanometer for parallel currents; column three for opposed currents; column four the calculated means of columns two and three; column five the deflections due to a single bobbin; and column six the differences between columns four and five.

The equation to the curve of column six is $y = \frac{A}{B^x} = \frac{470}{1.17^x}$, which is the same as the equation obtained by Jamin for the same case.

From the above table it may be seen that, when currents are parallel, the deflection of the galvanometer is greater than when they are opposed; and, when the current is sent through a single bobbin, the deflections are very nearly the means of the other two, as should be the case if Ampère's theory were true.

The conclusions which I have drawn are: —

1°. Parallel currents add to each other.

2°. Opposed currents diminish each other.

3°. The action in the latter case ought to be nothing at the middle of the bar. Experiment shows this to be the case.

Having thus shown the different results of these two methods, I purpose now to show that Jamin's results, instead of requiring a modification of Ampère's theory, are a direct consequence of that theory.

Let us, in approaching this subject, first see what would be the condition of the Ampèrian currents in a bar placed at right angles across the core of an electro-magnet. Suppose the electro-magnet to be placed vertically, and the bar horizontally on top of it. Suppose further that the current passes, in the part of the inducing coil nearest the observer, from right to left. Then, since the Ampèrian currents, in the core of the magnet, would tend, at the angles made by the core and the cross-bar, to induce currents in the cross-bar parallel to those in the core, we should have the Ampèrian currents, in the face of the bar towards the observer, flowing from below upwards in the part of the bar to the right of the core, and from above downwards in the other half; *i.e.*, a current in one direction about the core induces currents in opposite directions in the two halves of the cross-bar.

I have proved this experimentally by placing the coil of fine wire connected with the galvanometer, before spoken of as the secondary coil, at different points on this cross-bar. When the coil was placed to the right of the inducing coil, the deflection of the galvanometer was in one direction; when the coil was placed on the other side, the deflection was in the opposite direction: showing that, in the two halves of the cross-bar, opposite Ampèrian currents do actually exist. That this effect was not due to the direct action of the principal coil on the secondary, was shown by substituting a glass rod for the cross-bar. Let us now, keeping the two bars in the same relative position, make the cross-bar the core of two coils, and let the bar which we have just used as a core represent the armature used in Jamin's experiments. When opposite currents are sent through the primary coils, opposite Ampèrian currents will be induced in the two halves of the bar; and, as the converse of the preceding experiment, parallel currents will be induced in the armature, and these, strengthening each other, will increase the attraction between the bar and the armature. If, on the contrary, parallel currents be sent through the coils, parallel Ampèrian currents will be induced in the bar, and opposite currents in the armature; and, if the armature be placed at the middle of the bar, these currents should neutralize each other and the attraction ought to be nothing.

To prove these conditions of the Ampèrian currents experimentally, I have fixed the secondary coil on the armature at some considerable distance from the bar, and so investigated the conditions of the currents

in the armature, when currents were sent through the primary coils in the same and in opposite directions, with the following results. The armature being placed at the middle of the bar, and parallel currents being sent through the primary coils, there was no deflection of the galvanometer. When opposite currents were sent through the primary coils, the deflection was about 60 mm. A current through a single coil gave a deflection of 32 mm., or very nearly the mean of the other two. A very slight correction was made, due to the direct action of the primary on the secondary coil.

These experiments then gave the same results as those at which we had arrived theoretically, showing most conclusively that this is the correct explanation of Jamin's results.

As a further proof of the illegitimacy of Jamin's conclusions, and a proof which is independent of the secondary coil and galvanometer previously used, I have, using Jamin's apparatus, with the single modification of making the armature quite long in proportion to its diameter, and approaching it to the bar always in such a position that its longer axis shall be parallel to the axis of the bar, succeeded in obtaining results directly opposed to those of Jamin, and in harmony with the result of my previous experiments.

In order to make the experiment plain, let us see what ought to be the condition of the Ampèrian currents in such an armature.

Since it is quite long in proportion to its diameter, the Ampèrian currents would tend to arrange themselves at right angles to its axis; and, approaching the armature in the manner described, the currents in the armature would be parallel to those in the bar. With such an armature, therefore, we ought to have a greater attraction when the currents through the primary coils are parallel than when opposed.

That this is the case I have proved by the following experiment: A small armature of chemically pure iron was made, with a length of 6.5 mm. and diameter of only .8 mm. This was approached, with its axis parallel to the bar, always to the middle of the bar, since it is at this point that the difference in effect of parallel and opposed currents ought to be most marked.

The actual strength of the magnetism of the bar at this point could be varied by moving the inducing coils to or from the middle point, and in this way the intensity was made such that when parallel currents were sent through the coils the armature was supported by the bar. Upon reversing the current in one of the coils and again approaching the armature, the attraction of the bar was insufficient to support it in this position, although it would assume a position at right

angles to the bar, when, as should be the case, it was supported. These results show, as we had expected, that parallel currents increase each other, while opposite currents diminish each other.

The experiment was repeated, approaching the armature with its axis at right angles to the axis of the bar, when results similar to those of Jamin were obtained.

The delicacy of these experiments required a great number of repetitions. This was done, and care was also taken to carefully clean the armature each time lest any moisture from the hands, or other foreign matter, should make it adhere to the bar. Chemically pure iron was used to prevent the armatures acquiring a permanent magnetism.

I have thus attempted to show that the results of M. Jamin's experiments, although undoubtedly correct, do not warrant the conclusions respecting Ampère's theory which he has drawn, but, on the contrary, are a direct consequence of that theory: first, by investigating the condition of the currents in the armature; and, secondly, by showing that contrary results are obtained by making the armature very long in proportion to its diameter, and approaching it always with its longer axis parallel to that of the bar.

XXI.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

VIII.—AN EXPERIMENTAL PROOF OF THE LAW OF INVERSE SQUARES FOR SOUND.

By William W. Jacques.

Presented, May 10, 1876.

There is every dynamical reason for believing that the intensities of light, heat, and sound, diminish as the reciprocals of the squares of the distances from their origins.

That this is true of light and heat has been demonstrated experimentally. The case of sound, however, has only been put to the test in experiments so crude as not at all to warrant the assumption of the law on experimental grounds.

The following method (which was suggested by the reading of Professor Mayer's paper in the "American Journal" for January, 1873) ranks in its degree of accuracy with those which have been applied to the verification of this law in the cases of light and heat. It depends, primarily, on the principle, that, when a particle of air is solicited by two equal and opposite forces, it will remain at rest.

If two resonators, adjusted so as to resound with equal intensity, be placed equally distant from an organ-pipe, and connected by tubes with the two prongs of a fork-shaped tube in such a way that the sound-wave from one resonator shall arrive at the fork in opposite phase to that from the other, we shall have this condition; and, if the stem of the fork be placed in the ear, no sound will be heard. If, in place of one of these resonators, we put two, each of the same intensity as the first, both connected with the same prong of the fork, we may, by moving them farther from the source of sound than was the single resonator, at the same time altering the length of the tubing so that the wave from the single one shall arrive at the fork in opposite phase to that from the pair, produce the same effect of complete interference. If the law of inverse squares holds true, the distances of the single resonator from the source of sound should be to the cor-

responding distance of the pair as $1:\sqrt{2}$. If three resonators be opposed to one, the distances should be $1:\sqrt{3}$; if four resonators, $1:\sqrt{4}$ or $1:2$, &c.

It will be seen that the accuracy of the above method depends upon the following conditions:—

1st, That the resultant wave from the combination of two resonators has twice the intensity of that coming from one.

2d, That the decrease in intensity of a sound, in passing through a tube, is inconsiderable.

3d, That the intensity of resonance is proportional to the intensity of vibration at the mouth of the resonator.

Let us first see the arrangement of the apparatus used, and then determine how nearly these three conditions are satisfied.

As a source of sound, a C_3 closed organ-pipe was used, blown by a stream of air from a large gas-holder having an arrangement for keeping the pressure constant. The pipe was mounted on a small standard, raised some four feet above the floor, so that the sound-waves produced might have opportunity to diverge equally in all directions. At a measured distance from the embouchure of the pipe were placed two resonators, each cylindrical in shape, and capped with a hemisphere at one end, through which ran a tube ¼-in. in internal diameter, and at the other end with a flat plate, in which was a circular aperture of 1.5-inch diameter. The resonators were telescoped, so as to be readily adjusted for pitch and intensity. From the small tubes of the pair of resonators pieces of rubber tubing led to the two prongs of a forked brass tube, in which the waves from the two resonators came together, and augmented each other. From the stem of the fork another tube led to one arm of the trombone interference apparatus of Herschel. The third resonator was placed at an appropriate distance from the embouchure of the pipe, and so arranged that it could be moved to or from the pipe, and, at the same time, one arm of the interference apparatus could be moved to compensate for the change in phase due to such motion. From the small opening of this resonator a rubber tube extended to the other arm of the interference apparatus; and in this tube was inserted a brass fork precisely like that used for the pair of resonators, excepting that one of its arms was stopped, so that the conditions of reflection for this wave might be as similar as possible to those from the pair of resonators. If all these conditions of reflection be the same, it follows that two resonators give twice as great an intensity as one placed at the same distance from the source of sound.

The second condition, that the decrease in intensity in passing through a tube is inconsiderable, is abundantly proved by the experiments of Biot and Regnault in water-pipes. The third condition, that the intensity of resonance varies directly with the intensity of vibration of the air just outside of the resonator, seems not to be susceptible of experimental proof, excepting on the assumption of the law of inverse squares.

There seems, however, to be no cause for any considerable variation from this ratio: and if, upon trial, we find that the law does hold, it is reasonable for us to conclude that the variation of resonance is proportional to the intensity; for it is extremely improbable that there would be two errors which would exactly counterbalance each other.

The resonators having been adjusted so as to resound with equal intensity by comparing them two at a time on the interference apparatus, it was only necessary to connect three resonators as described above, so that the resultant wave should act on the air contained in the tube which enters the ear. Keeping now the pair of resonators in a constant position, and moving the single resonator and one arm of the interference apparatus until the resultant sound is at its minimum intensity, the relative distances should be $\sqrt{2}:1$.

Below are given several series of readings of the distance of the single resonator from the source of sound. The first three columns are the results of experiments made in front of the pipe, the pair of resonators being placed at a distance of 142 cms. from the embouchure. The resonators were lettered, for convenience, A, B, and C; and the three series of readings are the results of opposing successively A to B and C, B to A and C, and C to A and B. In the last three columns are given the results of similar measurements behind the pipe, the embouchure being still taken as the source of sound, and the pair of resonators being distant 177.5 cms.

At the bottom of the table the means are compared with the calculated positions of the single resonator.

The mean of the means of the first three columns is 100.3 cm; which differs from the theoretical by only 0.3 cm. The mean of the last three is 128.2, giving a difference from theory of 3.2 cm.

An inspection of the following table shows us, that, assuming the embouchure of the pipe as the source of sound, in front of the pipe the law of inverse squares holds almost exactly true: behind the pipe there is a slight difference. Theoretical considerations of the way in which the sound-waves are given off from a closed pipe would lead us to expect an error of this kind. The error due to the experiments

being conducted in a hall was probably inconsiderable, as the hall was 92 feet long and 65 feet wide; and, moreover, the windows were partially open. It should be remarked, that bringing the resonators too near the pipe introduced an error of a nature and magnitude which indicated that for a sound of considerable intensity the resonance was not proportional to the intensity of sound at the mouth of the resonator; but this exception only serves to prove the rule for the case of moderate intensity.

A opp. to B & C.	B opp. to A & C.	C opp. to A & B.	A opp. to B & C.	B opp. to A & C.	C opp. to A & B.
cm.	cm.	cm.	cm.	cm.	cm.
99	102	100	128	128	128
100	100	99	129	127	131
100	100	103	129	128	133
101	100	99	129	128	131
99	102	101	125	127	132
99	99	99	127	129	127
100	100	104	125	129	128
101	103	100	127	129	129
100	104	100	126	129	126
98	103	100	129	128	126
101	100	101	128	129	127
99	100	100	128	127	127
99	101	100	128	129	128
100	101	99	128	128	129
98	100	101	128	126	124
100	102	100	132	124	129
Mean = 99.6	Mean = 101.0	Mean = 100.4	Mean = 128.4	Mean = 128.5	Mean = 127.8
Theor. = 100.0	Theor. = 100.0	Theor. = 100.0	Theor. = 125.0	Theor. = 125.0	Theor. = 125.0

It is true that these experiments do not furnish an *exact* proof of the law of inverse squares for sound; but we have not an *exact* proof of the same law in the cases of light and heat. All that we can say of any of them, on experimental grounds, is, that they are *very* approximately true.

The above experiments show that we may make this assertion for sound on as valid experimental grounds as for light or heat.

IX. — DIFFRACTION OF SOUND.

BY WILLIAM W. JACQUES.

Presented, May 10, 1876.

THE following experiments were made in order to test the possibility of applying to our atmosphere the principles of Fresnel and Huyghens, which, in their application to the ether, have been attended with such fruitful results.

There seems to be no *a priori* reason why the particles of air, forming, as they do, a medium which, so far as the transmission of wave motion is concerned, is essentially similar to the ether, should not be so acted upon as to produce the interferences known in optical science as diffraction fringes. The following experiments bear upon this point.

The apparatus was so arranged, in the first course of experiments, as to give the best conditions for the study of external fringes, or those produced outside of the geometric shadow of a sharp edge, on which sound waves, diverging from a centre, were allowed to fall.

In the second course, similar waves were made to impinge upon an isolated narrow obstacle, and so to give rise to a system of interior fringes.

All other phenomena of diffraction may be classed as particular cases of one or the other of these two kinds.

First series. A board one hundred and fifty centimetres wide was placed at right angles to, and in contact with, the side of the hall. One hundred and fifty centimetres from the edge, in a line at right angles to the plane of the board, was placed a B_4 stopped lead organ pipe. On the other side of the board, a system of co-ordinates was established by means of light wooden rods, running parallel and perpendicular to the board; these rods being divided into centimetres, it became very easy to locate the points of interference by referring them to these co-ordinates, and so to trace out the bands of interference.

In order to distinguish these bands, I first tried applying my ear to different points along one of the rods; but they were not sufficiently well marked to make them apparent to the unaided ear. I then arranged a resonator, of proper size to resound to the pipe, to slide along the rod; connecting this, by a piece of firm rubber tubing, with the canal of my ear. The other ear I filled first with cotton, and then with putty; so that it was entirely deaf to all sounds. I was thus en-

abled to annul the effect of every sound but that which was re-enforced by the resonator. Moving now the resonator along the rod, I was able distinctly to mark points of maximum and of minimum intensity, which, in general, coincided with the positions which these bands should occupy as calculated by formulæ essentially similar to those applied in the diffraction of light.

In the first experiments, a number of quite serious difficulties were encountered. Perhaps the most important was that due to the fatigue of the sense of hearing, in consequence of which a maximum was estimated before it actually occurred. By alternately opening and closing the mouth of the resonator with the finger many times in quick succession, however, and by taking a reading by first moving the resonator in one direction, and another by moving it in the other, it became possible to set it with considerable accuracy, the differences being, generally, only a few centimetres. Another difficulty was met with in the shape of a distinct band of interference, seeming to have no connection whatever with the other bands, and following quite a different law. Upon tracing it out, however, it was found to be due to an interference of the direct wave with the wave reflected from the gas-holder used to blow the pipe. Upon covering the holder with a cloth, this was very much diminished: and, upon removing the holder, the interference band disappeared altogether. There seemed to be slight evidences of nodes and loops formed in the hall, as in an organ pipe; these, however, were very indistinct indeed, and were probably not a source of error. An attempt was made to use the manometric flame and revolving mirror to determine the points of most complete interference, but the difference in effect upon the flame was so slight as to render this method entirely impracticable.

In fact, the phenomena of diffraction can be studied only by the closest attention with the ear, and the ear is certainly far more delicate than any instrument of this kind that has ever been constructed. A very pure note, and one of constant intensity was necessary for the best results, and these were obtained by blowing the pipe with a stream of air from a gas-holder, having an arrangement for keeping the pressure constant.

Second series. The same board was set up near the middle of the hall. Two hundred centimetres from its middle point, on one side, was placed the pipe used in the preceding experiments. On the other side was arranged a system of co-ordinates within the sound shadow. The method of determining the points of interference was the same as in the first series; excepting that only the bands of minimum intensity

were noted. The whole phenomenon was less distinctly marked than in the case of a single edge; and the bands of maximum intensity were not definitely recognizable. It was only with careful attention that even the bands of minimum intensity could be discovered.

Below are given tables showing the observed and theoretical co-ordinates of the points of interference noted. Table I. is the case of diffraction from a single edge, and Table II. from a narrow obstacle.

In Table I., the first column gives the distances from the edge of the board measured perpendicularly to its plane: the second and third columns give the observed and calculated abscissas corresponding to these ordinates, for the points of maximum intensity noted; and the fourth and fifth columns the corresponding values for the points of minimum intensity.

In Table II., the first column gives the distances from the middle of the board; the second and third, the observed and theoretical ordinates of the first curve of minimum intensity; and the fourth and fifth columns, the ordinates of the second curve, — all to the right of the middle line; the sixth, seventh, eighth, and ninth give corresponding values for the two curves to the left of the middle line.

TABLE I.

	Curve of Max. Intens.		Curve of Min. Intens.	
Distances.	Obs.	Theor.	Obs.	Theor.
cm.	cm.	cm.	cm.	cm.
100	94	88	129	148
150	120	115	196	200
200	142	144	235	250

TABLE II.

Distances.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.
cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.
100	18	19	52	59	17	19	49	59
150	22	25	68	74	21	24	69	74
200	24	30	80	87	24	28	75	89

The theoretical values in the above tables were obtained by a graphical solution, as the formulæ, after undergoing the changes necessarily made, because of the sound waves being of considerable magnitude, became quite cumbersome, and extreme accuracy was not required. The value of λ was carefully determined and properly corrected for temperature.

It should be remarked that all of the above experiments were made in the large hall of the Institute, a room 92′ by 65′.

The phenomena of the diffraction of sound are not so distinctly marked as those of the diffraction of light. An examination of the tables, which are the results of a most careful series of observations, show that we are not warranted in accepting them as a basis for such excellent further work as has been done in the case of light. It is quite possible, of course, to calculate, from the positions of the fringes, the values of λ and therefore of V; to determine the temperature of the room in which the experiments are carried on; or, given these quantities, to deduce the values of physical quantities which are intimately connected with the propagation of sound, and to determine acoustic quantities analogous to those similarly deduced in physical optics: but the method is difficult, uncertain, requires the use of a large hall, physical annoyance to the observer, and, above all, is not susceptible of the desired degree of accuracy.

Their chief value seems to lie in their reactive effect on physical optics. In acoustics we are *sensibly* aware that we are dealing with waves propagated in an elastic medium. These waves may be felt and even seen.* Finding similar effects in optics to those here observed, we immediately refer these similar effects to similar causes, and so place our explanations of the diffraction of light and of the various cases of ethereal interference on a much firmer basis. The experiments show, too, that the principles of Fresnel and Huyghens, announced for the ether, are also applicable to our atmosphere.

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* Expts. of Topfer. Pogg. Ann. 1867.

X.—COMPARISON OF PRISMATIC AND DIFFRACTION SPECTRA.

By Professor E. C. Pickering.

Presented, June 9, 1875.

The object of the present communication is to afford a means of comparing the advantages of the two methods commonly employed for producing spectra, by diffraction gratings and by prisms. Two questions at once present themselves, the comparative length or dispersion, and the comparative brightness of the spectra. In adopting a standard of comparison, it is evidently necessary to select an absolute unit, which shall be wholly independent of the instrument employed, and defined entirely by the ordinary units of distance and direction. In comparing the two kinds of spectra, since an observing telescope and collimator are employed in both, it will be best first to compare the effect of the prisms and gratings alone, and then see how far both are affected by the telescopes. In the case of a diffraction grating, if i is the angle of incidence, r the angle of reflection, D the distance between the lines, λ the wave length, and n the order of the spectrum, these four quantities must be connected by the relation $n\lambda = D(\sin i + \sin r)$. The dispersion or angular deviation of two rays whose wave length differs by $d\lambda$ is found by differentiating r with regard to λ, recollecting that i being constant, its differential equals zero. We thus obtain $nd\lambda = D \cos r dr$, or $\frac{dr}{d\lambda} = \frac{n}{D \cos r}$. If now the grating is placed at right angles to the observing telescope, as in Meyerstein's spectrometer, the dispersion takes the very simple form $\frac{n}{D}$, or is independent of the angle of incidence and of the wave length, and hence is uniform throughout, and is simply proportional to the order of the spectrum, and inversely as the distance between the lines. This position has the further advantage that it gives a minimum of dispersion, and that consequently a slight error in setting is unimportant. If N is the number of lines per millimetre, or equals $\frac{1}{D}$, the dispersion assumes the still simpler form nN. This, then, forms the proper term of comparison for diffraction gratings, or the length of any minute portion of the spectrum will be proportional to its order, and to the number of lines per millimetre.

As an example, the grating that Angström employed in most of his measurements contained about 133 lines to the millimetre; and, as he commonly observed the fifth or sixth spectrum, his dispersion equalled 665 or 798. The admirable gratings of Mr. Rutherford contain 6480, 8640, 12,960, and 17,280 lines to an inch, or 255, 340, 510, and 680 lines to a millimetre. Accordingly the fifth spectrum of the 8640 grating would have a dispersion of 1·700.

The case of refraction is a little more complex. As shown elsewhere (Proc. Am. Acad. vii. 478), when a beam of light, having an index of refraction n, passes through a prism having an angle a, we shall have the relation $\frac{dr_2}{dn} = \frac{\sin a}{\cos r_1 \cos r_2}$, in which r_1 and r_2 are the angles of refraction after passing the first and second surfaces. For the position of minimum of deviation, $r_1 = r_2 = \frac{1}{2} a$, and in this case $\frac{dr_2}{dn} = \frac{2 \sin \frac{1}{2} a}{\cos i} = \frac{2}{n} \text{tang } i$. If, as is commonly the case, $a = 60°$, $\frac{dr_2}{dn} = \sec i$. But this gives $\frac{dr}{dn}$, while we want $\frac{di}{d\lambda}$, which may be obtained by multiplying by $\frac{dn}{d\lambda}$. The latter may be deduced from Cauchy's formula, $n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$. Differentiating this equation, $\frac{dn}{d\lambda} = -\frac{2B}{\lambda^3} - \frac{4C}{\lambda^5}$, and multiplying by $\frac{dr_2}{dn}$, gives $\frac{dn_2}{d\lambda} = \frac{-4}{n\lambda^3} \text{tang } i \left(B + \frac{2C}{\lambda^2}\right)$, or for a $60°$ prism $-\frac{2}{\lambda^3}\left(B + \frac{2C}{\lambda^2}\right) \sec i$.

The substances most commonly used for spectroscope prisms are flint glass and bisulphide of carbon. The indices of refraction of the first of these varies very greatly with the composition, and that of the second with the temperature. The lines B, E, and G are selected as showing the effects of the ends and central portion of the spectrum. The indices for flint glass are those given by Fraunhofer for the specimen No. 23, and equal 1.62775, 1.64202, and 1.66028. For the bisulphide of carbon the temperature of 11°5 C. is employed, and the indices 1.6207, 1.6465, and 1.6886. These values give for the flint glass, $B = .00789$ and $C = .000307$. The corresponding values for the bisulphide are, $B = .00614$ and $C = .001972$, the wave lengths being expressed in thousandths of a millimetre. From these we may compute the three values of $\frac{dn}{d\lambda}$ for flint glass to be .0568, .1381, and .2804; and for bisulphide of carbon, .0818, .2293, and .6073. And finally multiplying these values by 1000 to change the unit from thou-

sandths of a millimetre to millimetres, and by $\frac{dr}{dn}$, gives the following values for the dispersion of a 60° flint glass prism. For B, 98; for E, 242; and for G, 503. The corresponding values for a 60° prism filled with bisulphide of carbon are: for B, 140; for E, 404; and for G, 1133. Comparing these numbers with those given above for diffraction gratings, we see the superiority of the latter as regards dispersion, especially at the red end of the spectrum.

These advantages are, however, in a measure counterbalanced by the greater loss of light. It is shown elsewhere (Am. Jour. Sci. xlv.) that in a spectroscope containing ten 60° prisms the loss of light by reflection would equal 50.9 per cent; so that the transmitted ray would have an intensity of 49.1, the incident ray being taken as 100. This would be further reduced by the loss from absorption, but the amount would vary with the material, the wave length, and the length of path, or size of prisms. Estimating this loss as one half, still leaves the intensity of the whole of the spectrum as 25 per cent of the original beam passing through the slit. In a diffraction spectrum the light is much less; allowing one half for the light lost in the central white image, evidently if we have five spectra on each side, the average amount of light in each cannot exceed five per cent. And even this must be diminished by the loss due to reflection and absorption in the case of glass gratings, and to imperfect reflection in the case of speculum metal or silvered glass.

The discussion of the effect of the collimator and observing telescope on the dispersion involves another consideration; namely, the size of the image of the slit. To render this clearer, suppose we are observing the sodium spectrum, when a small amount of the metal is present.* We shall then obtain two sharply defined images of the slit separated by an interval dependent on the dispersion. These images may overlap, and will vary in width as the slit is open or shut, but their distances apart will not alter. Call w the true width of the slit, W that of its image as seen through the telescope, and referred to the distance of distinct

* If much sodium is present, the lines widen and become hazy, and with a large dispersion both appear again double, owing to the absorption of the outer layer of sodium vapor. This effect is readily obtained by putting a lump of borax in the flame, when at first it gives a bright blaze and shows the four lines; presently, however, the light becomes feeble, and the usual double line is alone seen. If now an image of the flame is projected on the slit, the spectrum of any part of it may be studied, and it will then be found that the central portion only gives the four lines, the edges giving the usual double line.

vision, 250 mms., or ten inches. Also call c the focal length of the collimator, o that of the observing telescope, and e that of a lens equivalent to the eye-piece. Then $W = \frac{250\,wo}{ec}$. Again, the dispersion or interval between the two images will equal that of the prism or grating, multiplied by the magnifying power of the observing telescope, or $\frac{o}{e}$, and will be quite independent of the collimator.

As in the microscope and telescope the highest powers are by no means those which give the best results, so in the spectroscope the best effects are not always obtained with the greatest dispersion. Increased angular dispersion is readily obtained by using a high power with the observing telescope, but the limit is soon reached, since the apparent width of the slit, and the various distortions, are increased in the same ratio. Moreover, with a very great dispersion the light is so far enfeebled that the spectrum becomes faint and the slit must be opened wider.

The most satisfactory test of the efficiency of a spectroscope, as of other optical instruments, is to examine some delicate object and compare the appearance with that obtained with other similar instruments. Formerly the D line was used for this purpose, which with any but the smallest instruments is seen to be double. In the solar spectrum it was soon found that there was a third line between these, and afterwards several other lines were noticed. The observations of Professor Cooke (Proc. Am. Acad. viii. 57), however, showed that most of these lines were due to the aqueous vapor in the earth's atmosphere, and that their visibility therefore depended very largely on the condition of the air. The E line is free from this objection; and, as it contains many more components, it furnishes a much more complete test. The following table gives the appearance of the E line as seen with various instruments. A dash denotes that the line opposite which it is drawn was visible. When a double line is seen as single, one of its components only is marked. The lines given in the map of Kirchhoff are shown in the column headed Kir. Those given by Angström are, in like manner, marked Ang. To obtain the lines seen with various instruments of the largest size, I asked several friends to draw all the lines they could see with their spectroscopes; and I take this occasion to express my thanks to them for the results. The column headed Sh. gives the results obtained by Mr. Sharples, with a large spectroscope belonging to Dr. Gibbs, having six 60° prisms, filled with chemically pure bisulphide of carbon.

The dispersion, therefore, as shown above, would be about 2400. Column Am. gives the lines seen by Dr. Amory, with a diffraction grating of Mr. Rutherford's, having 510 lines to a millimetre. As he employed the sixth spectrum, the dispersion was 3060. For the most recent and complete measurement I am indebted to Professor Young, who has measured the E line with a very perfect grating by Mr. Rutherford, having 340 lines to the millimetre ruled on silvered glass. As he used the eighth spectrum, the dispersion was 2720. These results have been taken as a basis, and the resultant wave lengths are given in the second column. My own observations are given in the column marked P., and were made in 1869 and 1870, with a spectroscope in which the light traversed each prism twice, giving a dispersion equivalent to 7, 9, or 11 flint glass prisms. This would correspond to dispersions of 1700, 2200, and 2640; but the best results were obtained with the two lower powers. All the observers used telescopes about a foot and a half in length.

We have thus four entirely independent maps, as neither observer had at the time a copy of the work of any of the others. The similarity of the results, with instruments differing so greatly in form and power, seems to show that we have nearly reached the limit beyond which an increase of dispersion is unadvisable; and as if with our largest instrument nearly all the lines really present in the spectrum were visible. It is much to be desired, however, that these lines may be compared with any other instruments of greater power, if such are ever constructed. It is only essential that the measurements should be made *before* comparison with the above results, since with the lines, as with faint stars, it is much easier to detect them when we know exactly where to look. Various tests may be selected from these lines for an instrument of any power. Thus to double the *E* lines, or to show 24 and 26 as two lines, is a good test for a one prism spectroscope of large size. The five pairs, (27, 28), (29, 30), (31, 32), (35, 36), and (37, 38), also form an excellent test for any but a very large instrument. The great number of double lines in this group, and more particularly in the *B* line, and in the electric spectrum of sodium, seems to prove most conclusively, as in the case of double stars, some real relation between the two components.

The last two columns give the relative intensity and width of the lines as estimated by Professor Young. Nos. 2, 16, and 27 are hazy, and No. 35 is a mere shade. The numbers under the observers' names give the approximate dispersion employed by each.

No.	Wave length.	Kir. 1400	Ang. 800	Sh. 2400	Am. 3060	P. 2000	Y. 2720	Intensity.	Width.
1	5260.9	—	—	—	—	—	—	3	2
2	1.3	—		—	—	—	—	6	3
3	1.8			?	—	—	—	1	1
4	2.1						—	3	1.5
5	2.4	—	—	—	—	—	—	5	2.5
6	2.8			—	—	—	—	1	2 5
7	3.2						—	2.5	1.5
8	3.4	—	—	—	—	—	—	6	2
9	4.0			—	—		—	2	2
10	4.4			—	—	—	—	1.5	1
11	4.8			—		—	—	2	1.5
12	5.0	—	—	—	—	—	—	2	1.5
13	5.1			—		—	—	2	1.5
14	5.3			—	—	?			
15	5.6			—					
16	5.8	—	—	—		—	—	7	3 5
17	6.3						—	1	1
18	6.5			—	—	—	—	1	1
19	6.6						—	.5	1
20	7.0			—		—	—	1.5	1
21	7.5	—	—	—	—	—	—	2	1.5
22	7.7			—		—	—	1.5	1
23	8.3						—	.5	1
24	8.7	—	—	—	—	—	—	10	3.5
25	9.0			?		—	?	.5	2
26	9.5	—	—	—	—	—	—	8	2
27	69.9						?	.7	
28	70.3						—	.7	1
29	0.5			—	—	—	—	1.5	1
30	0.7			—		—	—	1	1
31	1.2			—	—	—	—	2	1.5
32	1.6			—		—	—	2	1.5
33	2.3			—		—	—	6	2
34	2.5	—	—	—	—	—	—	5	1.5
35	3.1						—	1	3
36	3.9			—	—	—	—	.7	1
37	4.2			—		—	—	1.5	1
38	4.4	—	—	—	—	—	—	2	1.5
39	5.0			—		—	—	2.5	1
40	5.3	—	—	—	—	—	—	4	2

IV.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

XI. ON THE EFFECT OF TEMPERATURE ON THE VISCOSITY OF AIR.

By Silas W. Holman.

Read, June 14, 1876.

The developments of the "kinetic theory" of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.

According to the kinetic theory, the molecules of the gas are constantly in rectilinear motion. In virtue of their mass and velocity, these molecules have a certain momentum. Hence, if we have two layers of air moving over each other, we shall have a mutual interchange of momentum from the transference of molecules from one layer to another, the result being a tendency toward an equalization of the velocities of the two layers. This produces the effect of friction between the two layers, and its amount determines the viscosity of the gas in any particular case. From analytical considerations Maxwell has deduced* an expression which, as corrected by Clausius,† should read,

$$\eta = \frac{Mu}{4\pi s^2}$$

where η is the coefficient of viscosity of any gas; M is the mass of a molecule; u the "velocity of mean square" of the molecules; and s the dis-

* Phil. Mag. xix., xx.; 1860. † Phil. Mag. xix., 434.

tance between the centres of two molecules at impact. The value of η is expressed in units of length, mass and time, since it is a tangential force. This formula, if true, shows that the viscosity of any gas should be independent of its density at a constant temperature, and should increase proportionally to the value of u. But u^2 is proportional to the absolute temperature, whence we see that the viscosity should increase proportionally to the square root of the absolute temperature (which we may reckon from — 273°C.). Maxwell has also pointed out * that in this expression we should obtain the same result with regard to the pressure, whatever assumption we adopt of the mutual action at impact of the molecule; but that it is necessary to make some special assumption upon the nature of this action to determine the variation with the temperature.

Previous to this deduction by Maxwell, there had been but little work done upon the viscosity of gases, and almost nothing as to its variation with temperature. Subsequently, experiments have been made by Meyer, Maxwell, Puluj, and von Obermayer. The forms of apparatus used have depended upon two fundamental methods: 1°, the retardation of pendulums by the surrounding gases; 2°, the transpiration of gases through capillary tubes. In the present paper, I propose to discuss somewhat the value of these experiments in determining the variation of the viscosity with the temperature, and to describe some recent experiments made with a modification of the second of the above methods.

In a paper published in Poggendorff's Annalen, cxxv., 177, 1865, O. E. Meyer describes a series of experiments upon the internal friction of air made by measuring the retardation of three circular glass plates oscillating around a vertical axis in a closed receiver containing the gas, whose temperature and pressure could be varied. From the results of these measurements, Meyer concludes that the coefficient of viscosity is independent of the pressure. It will, however, be evident, upon an inspection of the published results, — especially by application of the graphical method, — that no reliance can be placed upon them for determining variation with the temperature. Meyer's second paper (Pogg. Ann. cxxvii., 199, 353) is devoted to a discussion of Graham's transpiration experiments,† from which we may derive quite a satisfactory proof of the law of Poiseuille as applied to gases. In the Philosophical Transactions, London, 1866, Maxwell published a series

* Phil. Mag. xxxv., 211.

† Phil. Trans. Roy. Soc. Lond. 1846–49.

of results obtained by a similar apparatus to that used by Meyer. From these Maxwell concludes that the viscosity is independent of the pressure upon the gas, and that it increases as the first power of the absolute temperature. If, however, the results published in that paper be all upon which this law is based, we cannot regard it as very securely established. A third paper was published by Meyer, in Pogg. Ann. cxliii., 14; in which the results of seven experiments with oscillating plates after Maxwell's pattern, but with bifilar suspension, were given. These, like the others, are insufficient to determine the effect of temperature. In three subsequent papers* by Meyer a large number of experiments are described. These were made by the method of transpiration through capillary tubes, and preliminary experiments were made to prove the validity of the law of Poiseuille. This law may be expressed by the following equation: —

$$V = \frac{\pi R^4 t}{8 \eta \lambda} \cdot \frac{p_1^2 - p_2^2}{2p} \qquad (1.)$$

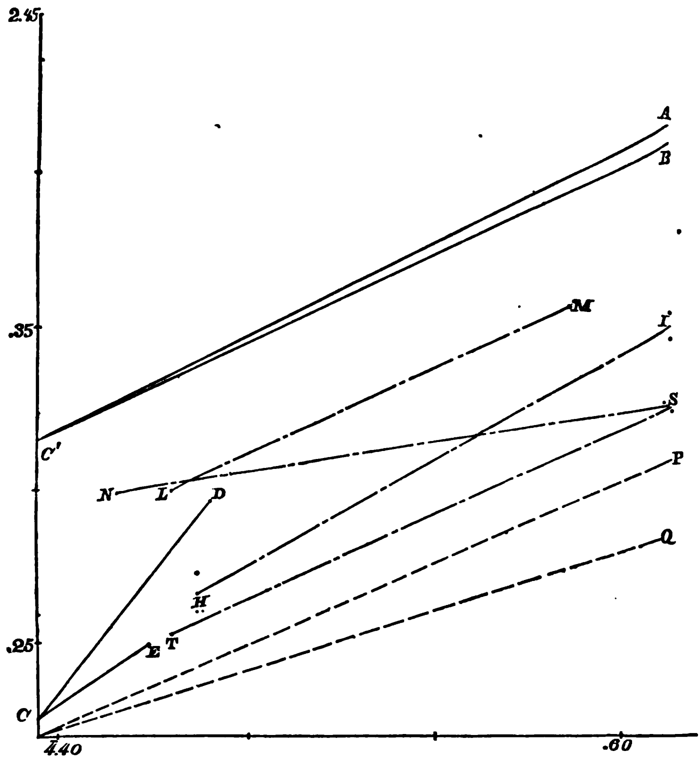
where V is the volume of gas transpired in the time t, measured at the temperature of the capillary, and under the pressure p; the pressure at entering the tube being p_1, and at leaving it p_2. The length of the capillary is λ, and its radius R; η being the coefficient of viscosity of the gas. This law may, I think, be regarded as established for variations of pressure not exceeding two atmospheres, and for tubes in which the length is very large as compared with the diameter.

Meyer gives a series of twenty-five experiments, and selects eleven as the most reliable. These all seem to indicate an increase of viscosity with rising temperature greater than the $\frac{1}{2}$ power, but appear at the same time quite discordant among themselves. Upon the accompanying figure, I have shown the extremes of these by a graphical representation. The method used to discuss them is one described in the Proceedings of the Academy for 1874, page 222. If we have a line of the general form represented by the equation $y = mx^n$, we may take logarithms of both sides and get the equation, $\log y = n \log x + \log m$, which has the form of the equation to a straight line. Hence, if we have the coördinates of a series of points which we suppose may be connected by a curve of the exponential form, we may determine this fact by plotting logarithms of these coördinates, which should give us points along a straight line whose tangent is the exponent in the primary equation. Thus, if our equation to the variation of η with the

* Pogg. Ann. cxlviii., 1, 203, 526.

absolute temperature τ be of the supposed form $\eta = c\tau^x$, where c is a constant, we may take the value of log η and log τ from our experiments, and expect upon plotting them to get a straight line making an angle whose tangent is x. This method I have applied to the results of Meyer, and the extreme points are shown at the points marked D and E in the accompanying cut.

Fig. 1.



The single experiment at zero centigrade gives the point C. A the other experiments furnish points scattered between D and E. Tl absolute values of the coefficient in these cases are: —

	$\tau =$	$\eta =$
C	273°.C	0.000168
D	293°.2	0.000198
E	287°.5	0.000178

For the line CD, $x = 2.3$; for the line CE, $x = 1.12$. This gives

an idea of the value of these results in determining the variation of the viscosity with the temperature. We cannot say from them, whether this variation is proportional to the first or second power of the absolute temperature. Even the results published in the fifth paper, which was to determine this law, are insufficient. In the first series of these results, shown upon the curve by the extreme lines *NS* and *TS*, we see that the exponent representing the law of variation with the temperature varies from $x = 0.21$ for line *NS* to $x = 0.69$ for line *TS*, a variation even greater than in the results previously discussed. All the other observations give points intermediate between *N* and *T*. The second series furnishes little better data; and the third series, from determinations with oscillating plates, are not sufficiently complete for discussion in this way. They, however, afford no greater satisfaction.

Puluj has used the method of transpiration for some measurements of this law, and his results appear in the Sitz'ber. Wien. Acad. of 1874, lxix., 287. The results which he has obtained appear rather more concordant than those of Meyer, but still show considerable disagreement. Upon the above cut, the lines *OP* and *OQ* show the extremes of these results as obtained by a discussion of his experiments. These lines do not represent the greatest variations between successive results in the same series, but the extreme variation between the mean results of various series. For *OP*, $x = 0.65$; for *OQ*, $x = 0.47$. It will thus be seen that these results are more concordant than the different series of Meyer: they are not, however, completely satisfactory.

Later than these we have a brief notice of some experiments by von Obermayer, in the Phil. Mag., xlix., 332, 1875, in which he states that he has obtained results "which confirm those of Meyer's experiments in a perfectly satisfactory manner." He states Meyer's results as furnishing the exponent $\frac{3}{4}$ for the variation of η with the absolute temperature; whence we must conclude that this number expresses the result at which he has arrived.

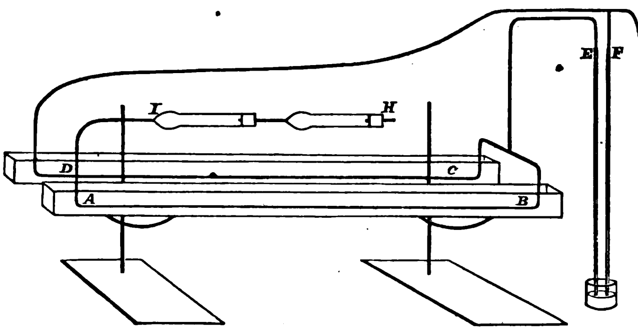
What value now are we to place upon these results, and which is the true one? Maxwell has given $x = 1$; Meyer, $x = \frac{3}{4}$; Puluj, $x = \frac{2}{3}$; von Obermayer, $x = \frac{3}{4}$. The first two values, $x = 1$ and $x = \frac{3}{4}$, we can hardly accept as certain, from the considerations previously shown. The value given by Puluj of $x = \frac{2}{3}$ is undoubtedly somewhat greater than is warranted by his results. Of the remaining experiments we cannot judge, since they have not yet appeared in full, so far as I have been able to ascertain.

The importance of this question in its bearing upon the kinetic

theory, as well as from its prominent place among the phenomena of gases, renders it very desirable that we should know the true law.

In endeavoring to arrange some new form of apparatus for a more accurate study of this law, the idea of a differential arrangement was suggested to me by Professor Pickering. This has been the origin of the following method. Two glass capillaries, *AB* and *CD*, were placed side by side, each in a tin trough to contain a bath to regulate the temperature of the gas. Air-tight glass and rubber connectors extended from *G* to the gauge *F*, and to the end *D* of one capillary. The ends *B* and *C* of the capillaries were connected with the gauge *E* by means of a T joint of glass. The end *A* of the second tube communicated with the external air through the chloride of calcium tubes *H* and *I*. The size of the connectors at the ends of *AB*

Fig. 2.



and *CD* was sufficient to allow the gas to assume the temperature of the bath. The tube at *G* was connected with a large flask, from which the air was continuously exhausted by means of a Richards' jet aspirator. The size of this flask rendered the pressure constant in spite of slight variations in water pressure. An inspection of this arrangement will show that when the flask is exhausted, and a vacuum produced at *G*, the air will enter at *A* under the atmospheric pressure, and will pass with constantly diminishing pressure to *G*; so that, at any intermediate point, as the junction of the two tubes at *BC*, we shall have a pressure intermediate between the two extremes. It will also be seen that the same volume of air is successively transpired through *AB* and *CD*; providing that there be no leak, which was carefully guarded against by making all the joints about *C*, *B*, and *E* — which were the only ones that affected the results — as tight as possible. By the two baths we may have the gas transpired successively through *AB* and

CD, either at the same or at different temperatures. Now, if we denote by V_1, R_1, λ_1, η_1, &c., the volume of gas transpired by AB, the radius and length of AB, and the coefficient of viscosity of the air passing through it, while V_2, &c., represent the same quantities for CD; also, if p_1, p_2, p_2, p_3 represent the pressure of the gas at A, B, C, and D respectively as obtained from the gauge and barometer readings; then from (1) we may write,

$$V_1 = \frac{\pi R_1^4 t_1}{8\eta_1\lambda_1} \cdot \frac{p_1^2 - p_2^2}{2p} \tag{2.}$$

and

$$V_2 = \frac{\pi R_2^4 t_2}{8\eta_2\lambda_2} \cdot \frac{p_2^2 - p_3^2}{2p} \tag{3.}$$

But if both baths are at the same temperature $V_1 = V_2$ if $t_1 = t_2$, and $\eta_1 = \eta_2$, whence we may write,

$$\frac{R_1^4\lambda_2}{R_2^4\lambda_1} = \frac{p_2^2 - p_3^2}{p_1^2 - p_2^2} \tag{4.}$$

Also in general it will be seen from the nature of the apparatus that $\frac{V_1}{1+a\delta_1} = \frac{V_2}{1+a\delta_2}$, where δ_1 and δ_2 represent respectively the temperatures at which V_1 and V_2 are transpired. Hence

$$\frac{\eta_1}{\eta_2} = \frac{R_1^4\lambda_2}{R_2^4\lambda_1} \cdot \frac{p_1^2 - p_2^2}{p_2^2 - p_3^2} \cdot \frac{1+a\delta_2}{1+a\delta_1} \tag{5.}$$

From equation (5) it will be seen that, in order to determine with this apparatus the ratio $\eta_1 : \eta_2$, between the coefficients of viscosity in the two tubes when the temperature of these is δ_1 and δ_2 respectively, we have only to know the ratio of the dimensions as expressed by $\frac{R_1^4\lambda_2}{R_2^4\lambda_1}$, and to measure p_1, p_2, and p_3 by reading three mercury columns. Also we can obtain a value of $\frac{R_1^4\lambda_2}{R_2^4\lambda_1}$ from readings of the gauges when $\delta_1 = \delta_2$, which needs only to be corrected for expansion of the glass to be used directly in equation (5). The whole process is thus reduced to the simple matter of reading columns of mercury, no measurements of volumes of gas being necessary. The nature of the correction of R and λ for temperature appears by putting into the above formulæ in which these values are supposed to be for 0° C, the coefficients of expansion of the glass $= A$; we thus get from (5): —

$$\begin{aligned} \frac{\eta_1}{\eta_2} &= \frac{R_1^4(1+A\delta_1)^4\lambda_2(1+A\delta_2)}{R_2^4(1+A\delta_2)^4\lambda_1(1+A\delta_1)} \cdot \frac{p_1^2 - p_2^2}{p_2^2 - p_3^2} \cdot \frac{1+a\delta_2}{1+a\delta_1} \\ &= \frac{R_1^4(1+A\delta_1)^3\lambda_2}{R_2^4(1+A\delta_2)^3\lambda_1} \cdot \frac{p_1^2 - p_2^2}{p_2^2 - p_3^2} \cdot \frac{1+a\delta_2}{1+a\delta_1} \end{aligned} \tag{6.}$$

Lest, however, an error might occur in the last reduction from a difference between the coefficient of expansion of the bore of a capillary tube and of its lineal expansion, I have carefully measured both, and find that the coefficient for the bore is 0.0000075, while for the linear expansion I find 0.0000080 per degree centigrade, a difference too slight to affect the results in my use of it; I have thought it best to use the value 0.0000075 as it entered in the fourth power, while the other entered only in the first power. The tubes used have also been calibrated to insure the selection of those of uniform bore, and their dimensions have been accurately measured by mercury and a micrometer screw. The dimensions of the two tubes used in the experiments to be described, were, for tube No. I., $\lambda = 1272.3$ mm., $R = 0.1098$ mm.; for tube No. II., $\lambda = 1274.1$ mm., $R = 0.1115$ mm.

To make an experiment with this apparatus, it is merely necessary to start the jet of water and allow the exhaustion to proceed until the mercury columns in F and E have come completely to rest. Readings are then taken of the heights of these columns by means of a cathetometer from a steel scale placed beside the gauges. The reading of the barometer corrected for instrumental error gives the pressure at A. All these are reduced to the freezing point, and E and F are corrected for capillarity by the tables of Delcros. The temperature of the baths is also taken by thermometers in various positions in the troughs. This must be kept constant throughout the experiment, and I have, therefore, principally used the temperatures of melting ice and boiling water. In the experiments of which the following table gives the results, advantage has been taken of the four methods of checking the results of one experiment by another, by reversing the direction of flow of the air through the tubes and heating alternately, in each case, first one and then the other trough. In the table, column first gives the number of the experiment; column second, the direction of flow of the air, which entered at the tube whose number is first given and passed out from the other; columns three, four and five give the pressures at A, B and D respectively; columns six and seven show the temperatures in centigrade degrees of the baths around tubes I. and II. respectively; column eight shows the values of the ratio $\frac{R_1^4\lambda_2}{R_2^4\lambda_1}$ at different temperatures; column nine, the values of $\frac{\eta_1}{\eta_2}$, *i.e.* of η at the higher to η at the lower temperature; column ten shows the values of the exponent x in the equation $\eta = c\tau^x$. This is the quantity which it was the object of the experiments to obtain.

No.	Dir.	p_1	p_2	p_3	$T_{I.}$	$T_{II.}$	$\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$	$\frac{\eta_1}{\eta_2}$	x
		m. m.	m. m.	m. m.					
1	I.-II.	759.9	525.2	16.3	17°.0	17°.0	0.912		
2	,,	,,	549.3	17.1	17.0	47.5		1.083	0.799
4	,,	759.8	525.6	18 0	15.1	15.1	0.916		
5	,,	,,	584.4	18.9	,,	,,	0.921		
6	,,	765.7	550.9	18.6	17.8	17.8	0.934		
7	II.-I.	,,	490.7	17.7	17.5	99.0		1.212	0.776
8	,,	,,	491.2	17.6	17.5	99.5		1.206	0.755
9	,,	,,	490.0	17.3	17.5	99.8		1.215	0.780
11	,,	755.2	467.8	20.4	0.0	100.0		1.272	0.771
12	,,	,,	468.4	19.4	,,	,,		1.267	0.757
13	,,	,,	467.9	19.6	,,	,,		1.271	0.768
14	,,	,,	467.7	19.3	,,	,,		1.273	0.773
16	,,	,,	544.2	20.7	0.0	0.0	0.927		
17	I.-II.	756.7	525.3	23.4	,,	,,	0.928		
18	,,	,,	594.8	21.5	0.0	100.0		1.277	0.782
19	,,	761.4	529.1	16.1	100.0	100.0	0.933		
20	,,	762.0	530.2	16 7	,,	,,	0.937		
21	,,	763.1	452.2	18.5	100.0	0.0		1.259	0.738

In the calculation of the ratio $\frac{\eta_1}{\eta_2}$ of this table, the value of $\frac{R_1{}^4\lambda_2}{R_2{}^4\lambda_1}$ used was the mean of that obtained from experiments 16 and 17, after correcting for temperature. The agreement of these two values within 0.1 per cent is a test of the accuracy of the method, as the two experiments were made on different days, and the direction of the current was reversed. It will be seen that the value of this quantity increases slightly with the temperature, as we should expect from the slight difference in size of the two tubes used. The values of x will be seen to agree quite closely, with the exception of experiments 2 and 21. I have treated these results in the same manner as those of Meyer, and the result is shown on Fig. 1.

The point A is plotted from experiment 18, and B from 21; so that the lines AC' and BC' show the greatest variation in nine out of ten determinations, while the majority of these lie so close together as not to be capable of clear representation between A and B. The point C' has been raised from C for distinctness. Experiment 2 would indicate a deviation from the straight line; but I do not regard this as a perfectly reliable determination. More experiments are needed between 0° and 100° to establish the law.

In order to compare these results with those of Meyer, I have been obliged to assume his value of $\eta = 0.000168$ at 0.° C. as a starting-point, since the apparatus which I have used does not give absolute values of the coefficient of viscosity, but only ratios. It would appear, however, that the great concordance among the results thus far obtained would warrant its application to absolute measurements, for

which it would only be necessary to measure the volume of the gas transpired in a known time. These, with experiments upon other gases, and also upon the validity of Poiseuille's law, I hope to be able to accomplish. The many points of superiority of this apparatus, and the excellence of these preliminary results, would seem to indicate more accurate determinations than others preceding them.

As a result of these experiments, it would appear that the viscosity of air increases proportionally to the 0.77 power, nearly, of the absolute temperature between 0° and 100° C. This value corresponds quite closely to the $\frac{3}{4}$ power, and we might infer that this was the value of x towards which the experiments pointed; but as I feel assured that further experiments will furnish still more concordant results, I should be unwilling to accept 0.75 until these had been performed. The general agreement of my results with the numbers of Meyer and von Obermayer would seem to point to the fact that the value of x cannot be as great as unity, and is probably about 0.75.

XIX.

MOUNTAIN SURVEYING.

By Professor E. C. Pickering.

Read, Jan. 11, 1876.

The difficulties and expense of a topographical survey are always very great; and this is particularly the case in a mountainous country, owing to the short horizontal interval between the contours, their irregularity, and the labor involved in reaching the more elevated portions. The objections to the usual trigonometrical methods are, that the theodolite, or transit, needed to measure the angles, is heavy, liable to injury when carried over a rough country, and the time required to measure each angle is considerable. The labor and cost of measuring a base-line are also very great. Moreover, the accuracy attained is much greater than is ordinarily needed; since, as the land is commonly of little value, there is no need of determining positions with more accuracy than they can be shown on a map. If the tract of country is large, a scale greater than $\frac{1}{100000}$, or $\frac{1}{50000}$, is rarely used; and, owing to the unequal expansion and contraction of the paper, long distances could not be measured with accuracy on such a map much nearer than within fifty to one hundred metres. Another objection to the trigonometrical method is, that the work must be carried on continuously from one base to the other; and no positions can be determined except by connection with a base through a series of triangles. If, however, the latitudes and longitudes of several points are ascertained, each of them may be used as a centre from which the form of the surrounding country may be determined; and an error in one will in no way affect the position of the others. The problem proposed, therefore, was to devise some instrument which should give approximately the distance and elevation of a mountain summit or other object, and which at the same time should be light and not easily injured. With such an instrument, an exploring party, whenever they camped at a point commanding an extensive view, could, during the

night, determine their latitude and longitude, and, during the day, locate all prominent visible objects.

To measure small distances, the method of the stadia and telemeter gives excellent results, but is open to the objection that an assistant is needed, who must carry a graduated pole to each point whose distance is to be determined. This method also is only applicable to short distances; as beyond five hundred, or at most a thousand metres, the pole appears so small, that its apparent length cannot be determined with accuracy.

These difficulties are, in a great measure, avoided in the following instrument. A good traveller's telescope, or spy-glass, is mounted firmly on a tripod, and a spider-line micrometer or scale of equal parts is inserted in its eye-piece. In front of the object-glass is fastened a piece of plane-glass, which may be set at any desired angle, and clamped firmly. The angle may be roughly measured by a small circle divided into degrees. The whole is free to turn around the axis of the telescope. To measure the distance of any object, D, Fig. 1, the angular magnitude of the divisions of the scale in the eye-piece, is first determined by the usual methods. The telescope is then mounted at A, and directed towards D, taking care to select for it some sharply defined object, as a rocky crag, the trunk of a tree, or the edge of a snow-bank. Select a second object, C, nearly at right angles to D, and turn the glass in front of the telescope until the reflection of C in its front surface shall be in the field at the same time as the image of D transmitted through the glass. Measure accurately the interval between the two images with the micrometer. Next measure off a distance AB from A towards C of one or two hundred metres, and place the telescope at B. Again measuring the interval between the two images, taking care not to disturb the mirror, a result will be obtained which will differ from the previous measurement by the angle ADB. From this triangle we deduce $DB = AB \frac{\sin A}{\sin D} = AB \sin A \frac{206265}{ds}$, in which d is the difference in the scale-readings, and s the magnitude of each division in seconds. A should be taken nearly 90°; in which case $\sin A$ will very nearly equal unity. Its value may be found with sufficient precision with the divided circle attached to the mirror, or by a plane-table. The greatest accuracy will be attained when AB has

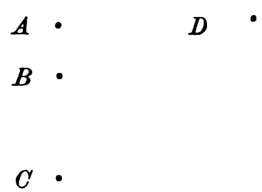


Fig. 1.

such a value as to make the angle at D nearly twice the diameter of the field. To determine the degree of accuracy attainable with such an instrument, suppose the diameter of the field of view 1°, and that an error of .1′ or 6″ is committed in the measurement, — a large error, considering how accurately seconds are determined with the spider-line micrometer in astronomical work. Suppose, again, that the object D is ten thousand metres distant, or 6.21 miles, and that AB is taken equal to two hundred metres: the error in question would then equal only fourteen metres, or forty-six feet, — a quantity quite insensible on the scale proposed above. Again: if the object is fifty or a hundred kilometres distant, it is only necessary to increase AB in the same proportion, and we shall still be able to measure the distance of D with the same proportionate accuracy, without yet using a base of inconvenient length. In this way, if the country is dangerous, the observer may measure the distance of all visible objects without going far from camp. Comparing this instrument with the stadia, we see that it has the advantage that it is not necessary to send a man to the point to be measured, and that the accuracy is the same as if he could carry a pole one or two hundred metres in length.

Three methods have been employed for the determination of heights. First, by the barometer. But this involves a visit to every point to be measured, and, at the best, is very inaccurate. Observations in Switzerland and California have shown, that with the best barometers, after applying all the known corrections, and even if each observation is the mean of thirty, taken once a day for a month at the same hour, at both the upper and lower stations, there still remains an uncertain error, amounting sometimes to two per cent of the whole height. How much greater, then, must be the error of a single reading, often made without simultaneous observations below, and with the defects of an aneroid added to the other errors! The most accurate method of determining a height is by levelling; but the labor and expense of this are too great to allow its frequent use in mountainous countries. The third method is that of zenith distances, which is largely used in the Coast Survey for determining heights. The altitude is here observed by a large vertical circle, which must be read with the utmost precision, since the angle, if the object is distant, rarely exceeds two or three degrees. It is claimed that at least an equal degree of accuracy may be attained by the instrument described below, while the expense of a graduated circle and delicate mounting is wholly avoided. The principle employed is that of the zenith telescope, so largely employed in determining the latitude. It consists

simply of the telescope, described above, turned around its axis 90°, and a delicate level screwed firmly to its tube. To make sure that the telescope is turned by precisely the right amount, it is well to have a second level at right angles to this to render the threads of the micrometer exactly horizontal. The size of the divisions of the level and of the micrometer must be previously determined; their relative value being most easily found by directing the telescope towards any distant object, and slightly inclining it, so that the bubble shall occupy various positions in the tube. The corresponding positions of the object are read by the micrometer, and a curve constructed with ordinates equal to these readings, and abscissas to the position of the middle of the bubble of the level. The reading of the micrometer corresponding to a perfectly level line must next be determined. This may be found by setting the telescope up at two not very distant points, and reading the height of each from the other. The mean will give the direction of a horizontal line; since the elevation in one case equals the depression in the other. The direction is, however, best found by observing the height of some known objects; since this eliminates various errors, as will be described below. The height of any object is more readily determined by directing the telescope towards it, and bringing the bubble nearly to the centre of the tube. Then read the position of the object by the micrometer; and, finally, read the exact position of the two ends of the bubble, taking care not to touch the telescope. These readings may then be reduced to seconds of altitude, as follows: Call A the required altitude in seconds, m the reading of the micrometer, m^1 its reading when the telescope is directed towards an object at the same height as its own, and b the mean of the two ends of the bubble of the level. Again, let s equal the magnitude of each division of the level in seconds, and l the corresponding magnitude of the level divisions. Then $A = (m - m^1)\ s + bl$. The elevation in metres or feet is then found by multiplying the tangent of this angle by the horizontal distance of the object, and correcting for the curvature of the earth and for refraction. The first of these corrections may be made with great precision by the formulas or table given in the Coast-Suryey Report for 1871, pp. 160 and 169. The second correction is, however, very irregular, and may, therefore, generally be regarded as nearly proportional to the square of the distance. Since the correction for curvature is also nearly proportional to the square of the distance, we may write the elevation $E = D \text{ tang } A + m\, D^2$, in which D is the horizontal distance, and m a quantity dependent on the condition of the air. If, therefore, the height of any distant object visible is

known, it is better to deduce m from its observed altitude, and from this compute the other elevations. Were it not for the uncertain error of refraction, this instrument would give results of extreme precision. Thus an error of 6″ in the altitude of a mountain one hundred kilometres distant would only correspond to a difference in height of about three metres. The uncertainty of refraction is much greater than this, and far exceeds the instrumental errors, except in a small telescope. Since, however, this cause of error is present when the theodolite is used, we see that altitudes can be obtained by this instrument with all the precision of the best theodolite; in fact, with all the accuracy of which the method is capable. Apart from its lightness and cheapness, it has this great advantage over a theodolite, — that, since the level is firmly attached to the telescope, there is little liability to error; while, as the theodolite measures the angle between the telescope and the horizontal limb of the instrument, any injury is liable to throw it out of adjustment. With the instrument as described above, no angles could be measured greater than the diameter of the field of view. This difficulty may be remedied by attaching another level, slightly inclined to the first, so that the two fields of view corresponding to a horizontal position of the two levels shall be nearly tangent to each other. A third level serves still further to extend the range of the instrument. Thus, if the field of view is about 2°, angles between 1° and — 1° may be measured by the first level, between 1° and 3° with the second, and between — 1° and — 3° with the third. The instrument, in this form, may be called a micrometer-level. One of its greatest advantages is the rapidity with which elevations may be measured. There is no difficulty in measuring thirty or forty mountains in this way per hour, without the labor of ascending them; while by the barometer it rarely happens that more than one can be measured in a day, and with the ordinary level the altitude of a high mountain would be the labor of days or weeks. The rapidity is also much greater than that of a theodolite; since no accurate mounting is needed, and a micrometer-scale can be read at least as quickly as the telescope can be set, so that the entire time of reading the circle by the vernier is saved.

One of the principal advantages of both the instruments here proposed is, that either may be made out of a telescope such as any explorer would be likely to carry. By simply adding a mirror in front, a photographed scale, and three levels, distances and elevations may be measured with all the accuracy ordinarily required. This method may be applied with especial advantage on a mountain-top;

since the elevations of all other mountains in sight, except those in the immediate vicinity, may be determined.

The fact is worth noting, that, even if the error from refraction could be eliminated, the method of zenith-distances would not equal in accuracy that of ordinary levelling. For suppose that the sights are taken at distances d, and that the probable error of each is e. If n sights are taken, or the distance travelled is nd, the probable error will be only $e\sqrt{n}$, since the positive and negative errors will probably in part neutralize. The error in the method of zenith-distances will, however, be proportional to the distance, or will be ne. Thus, if sights are taken every hundred metres with a probable error of 1 mm., the probable error of the level in ten kilometres will be 10 mms., since $n = 100$; while with zenith-distances the error would be 100 mms. Evidently, therefore, within reasonable limits, to attain the greatest accuracy with the level, the sights should be as short as possible, — a fact in accordance with general experience.

Evidently the telescope of a theodolite may be converted into a micrometer level by inserting in its eye-piece a scale, and attaching three levels. Small vertical angles, which are those most used in surveying, can then be measured more accurately than by a vertical circle. In the same way, a similar attachment may be made to the telescope of a plane-table; and the advantage is especially marked in this case, since an accurate mounting is not needed. A further application may then be made; namely, to determine the distance when the height is known. Suppose that a distant pond is observed from the top of a hill. The telescope is directed to various portions of its shore, and the apparent depression observed. Since every portion must be at an equal distance below the observer, it is easily shown that the distance is always inversely proportional to the depression. Accordingly, if the direction of each part of the shore is marked on the plane-table sheet by a line, and a distance is laid off on this inversely as the observed depression, a map of the pond is quickly made. This may be reduced to its true scale if we know the position of any one point, or the height of the observer above the water. If the shore is abrupt or wooded, only the farther edge of the shore can be thus surveyed, or rather the portion where the actual shore-line is visible. This method is in fact a form of stadia, in which the measuring-pole is replaced by the constant vertical distance between the eye and the plane of the water. The same method may be used for determining the form of an island, of a coast-line, or of a river winding through a nearly level meadow. The form of a pond or island may also be obtained in the same way

from a drawing made by a camera obscura or camera lucida, or from a photograph; and has the advantage that it begins to be accurate for depressions greater than 2° or 3° just where they pass out of the range of the micrometer-level as described above.

Valuable observations on the changes in the dip and in the refraction might be made with a large telescope of this form. It is much to be desired that such observations might be conducted for a period of years from two such stations as Mount Washington and Portland. As the pressure, temperature, and moisture of these points is already determined by the Signal-Service Department, a small additional expense would furnish a valuable addition to our knowledge of the atmospheric refraction.

XX.

HEIGHT AND VELOCITY OF CLOUDS.

By Professor E. C. Pickering.

Presented, Jan. 11, 1876.

The velocity of the wind at different altitudes is an important element in Meteorology, and the ordinary methods of measuring it are far from satisfactory. By the following method, it is believed that the velocity of the wind at considerable heights may be measured with an accuracy at least equal, and probably greater, than that of similar measurements near the surface of the earth. The apparatus consists simply of two similar camera obscuras formed of tripods covered with black cloth, and with cosmorama lenses above, which form an image of objects near the zenith on a sheet of paper placed beneath. A day is selected when cumuli clouds are crossing the sky, and the two cameras are placed at any convenient interval, as a hundred metres, in a direction nearly perpendicular to the direction of the wind. An observer with a watch is stationed at each camera, and when a cloud enters the field a signal is given, and each draws a line tangent to the edge of the cloud and parallel to the direction of the wind every half minute. At the intermediate quarter minutes, other lines are drawn perpendicular to these, and also tangent to the cloud. The first series of lines will be nearly coincident, the second at intervals marking the cloud's motion. The zenith is now marked on each drawing by suspending a plumb-line from the centre of each lens, or in some other way, and a line drawn through it parallel to the direction of the cloud's motion. It will now be found that the distance of this line from those parallel to it and tangent to the cloud is different in the two sheets by an amount equal to the parallax of the cloud, or the angle between the two cameras as seen from the cloud. The height of the cloud may then be easily determined, if we know the focal distances of the lenses and the interval between the cameras, by the proportion: Difference of distances of the two lines : focal length of lenses = interval between the cameras : required height of cloud. To determine the accuracy of this method, suppose the interval between the cameras one hundred

metres, the focal lengths one metre, and the height of the cloud one kilometre; then the difference between the distances of the two lines will be a decimetre. If this distance is measured with an error no greater than a millimetre, the height will be given within ten metres, or within one per cent. The velocity per minute is then readily deduced from the lines perpendicular to the direction of the wind, and the velocity of the latter may thus be determined within one or two per cent, a degree of accuracy at least equal to that of the best determinations of the velocity of the wind at the earth's surface and much greater than the degree of uniformity of any ordinary wind. Each cloud will furnish a measurement at a different height, and a comparison with observations at the surface of the earth will readily give the relative velocities at these various altitudes.

Various other applications of this principle will suggest themselves. For instance, if the paper is replaced by a sensitive photographic plate, and the cameras directed towards a distant thunder cloud at night, an image of each flash may be taken. A great many flashes may be recorded on each plate, and the corresponding images recognized by their forms. The distances and true dimensions may then be determined with considerable accuracy. If observations are made at the same time, of the interval between the flash and the thunder, the velocity of the sound may be measured, and it may be proved whether, as has been claimed, the velocity of such an intense sound is far greater than that of any ordinary noise.

[FROM THE AMERICAN JOURNAL OF SCIENCE AND ARTS, VOL. XI, JUNE, 1876.]

A NEBULA PHOTOMETER.

BY E. C. PICKERING.

AN examination of the article in the May number of this Journal on the changes on the Nebula M. 17 shows the desirability of accurate photometric measurements of these bodies. I wish therefore to make known the following nebula-photometer in hopes that some one having the use of a telescope of sufficient size may undertake such measurements. A plate ruled with squares is inserted in the eye-piece of an equatorial telescope with a minute circle of collodion near the center as in the photometer of Dove. It is illuminated in front by the nebula, and behind by a plate of glass inclined at an angle of 45° which reflects the light of a lamp placed on one side of the eye-piece. The light may be varied by two crossed Nicol's prisms, by passing the light through a slit whose width may be varied and measured, or in other ways. To measure the brightness of a nebula the various portions are brought in succession into the center of the field and the light varied until the spot disappears. The exact position of each point is found by observing the various positions of any star in the field with regard to the squares. The real motion of the photometer is thus found from the apparent motion of the star. A contour map may then be constructed showing the brightness of the various portions, and would soon show any marked changes in the light of the various parts. The light of the adjacent sky must be similarly measured and subtracted from all the readings. The brightness may be compared with that of any star by throwing the latter out of focus until its disk attains a given size, and a star photometer is thus obtained. Observations on a comet, with contours showing its brightness on various days, would be both interesting and valuable. The brightness of different portions of the moon could be measured by slightly modifying this photometer. By using a very low power the light of an aurora, of the zodiacal light or of different portions of the sky could be similarly measured. For very faint objects it might be better to insert a diaphragm in the eye-piece having an aperture but little larger than the collodion film, thus giving a dark background. Positions could then be determined by the finder or by moving the entire eye-piece by micrometer screws.

PROGRESS OF THE

PHYSICAL DEPARTMENT

OF THE

MASS. INSTITUTE OF TECHNOLOGY,

FROM 1867 TO 1877.

BOSTON, JAN. 31, 1877.

Prof. J. D. RUNKLE,

President of the Mass. Inst. of Technology.

DEAR SIR:—

My official connection with the Massachusetts Institute of Technology, which has extended over a period of precisely ten years, terminates to-day. I therefore take this opportunity of presenting a statement of the work I have accomplished, and of comparing the condition of the Physical Department and the nature of the instruction with that of ten years ago. At that time instruction in Physics was universally given by lectures and recitations illustrated by experiments and diagrams. A student wishing to pursue this subject further, might aid his instructor in preparing these experiments, and thus become qualified in his turn to deliver the lectures. The first step towards the introduction of a new order of things seems to have been taken by Prof. Wm. B. Rogers, then President of the Institute, and Professor of Physics and Geology. In a pamphlet entitled "Scope and Plan of the School of Industrial Science of the Massachusetts Institute of Technology," published in 1864, the following paragraph headed "Practice in Physical and Chemical Manipulations" occurs on page 23:—

"It will be the object of these exercises to make the student practically familiar with the adjustments and use of the apparatus and agents employed

in the more important experiments and processes in natural philosophy and chemistry. With this view, the students, under the direction of their teacher, will be called, by small classes at a time, to execute with their own hands various experiments in mechanics, pneumatics, sound, optics, electricity, and other branches of experimental physics, and to exhibit chemical reactions, to fit up chemical apparatus, to prepare gases and other products, and demonstrate their properties by suitable experiments, accompanying these manipulations, when required, with an explanation of the apparatus used, or of the process or experiment performed."

On page 24 the following paragraph occurs under the heading "Laboratory of Physics and Mechanics —

"In this laboratory, it is proposed to provide implements and apparatus with which the student may be exercised in a variety of mechanical and physical processes and experiments. Thus he may learn practically the methods of estimating motors and machines by the dynamometer, of experimenting on the flow of water and air, or other gases, and of testing the strength of the materials used in construction. He may become familiar with the adjustments and applications of the microscope; be practised in observing with the barometer, thermometer and hygrometer; and, in a room fitted up for photometry, may learn the mode of measuring the light produced by gas and other sources of illumination, and the value of different kinds of burners, lamps, and their appendages."

This appears to be the first clear statement of the desirability of teaching Physics by the Laboratory method, but owing to the pressure of other business and the lack of means, the matter stopped at this point, and no definite plan was formed for carrying it out.

Accordingly the first instruction in Physics at the Institute was given by the usual illustrated lectures, and this continued to be the case for the two following years. Meanwhile, my appointment in charge of the exercises in Physics led me to consider whether a plan for a Physical Laboratory might not be developed, and, accordingly, in April, 1869, a scheme was offered to the Government of the Institute and printed and distributed among them, under the title "Plan of the Physical Laboratory."

This plan was carried out the following autumn, and has been in operation ever since without sensible change, except in extending its scope. In fact, out of sixteen experiments published in 1870, all but one or two are still in use without alteration.

The cost of establishing this Laboratory was exceedingly small. The argument used was, "this is an experiment and may not prove a success, therefore we will begin on a small scale, and if it accomplishes its end the public will at once appreciate its value, and it will not be difficult to obtain a liberal support, while if unsuccessful, the loss to the Institute will be small." Here, if anywhere, the mistake was made; it accomplished far more than its projectors anticipated, but the public, seeing how much had been accomplished with so little, supposed that no more was needed. The great need of the Physical Laboratory at the present time is an independent endowment, by which, as regards apparatus, it may be placed on an equal footing with other Laboratories since established. Instead of an original expenditure of several thousand dollars, the cost to the Institute was little beyond that of fitting up the room adjoining the lecture-room with tables, and gas and water fixtures, and most of this was done by those already connected with the Institute. The apparatus was largely constructed in a similar manner, at small cost, and without regard to looks, the working being almost the sole consideration. Notwithstanding these adverse circumstances, the work of the Laboratory has been continually increasing. The space first provided proving insufficient, a portion of the long room at the end of the building was added, and soon after the whole of this room was appropriated to this purpose. Two large rooms, one nearly a hundred feet in length, were thus devoted to Laboratory work. But, as many small rooms were needed rather than a few of large size, two alcoves were partitioned off from the main room, one devoted to a chemical store-room, and the other fitted with tools for a workshop. Recently, a partition has been run along one side of the large room, by which small rooms are formed for electrical measurements, photometry and optical experiments requiring the exclusion of a portion of the light. The use of a portion of the closet adjoining the physical lecture-room has also been granted for a battery room, and for photographic work, and last year the room to the east of the front door has been darkened, so that it may be used for spectroscopic and other similar work. Notwithstanding this large amount of space, it is not of the kind best adapted to a Physical Laboratory. Accordingly last summer I

presented a plan by which the same facilities could be obtained with a much more economical arrangement of space. I proposed that the present rooms should be given up to lecture-rooms, or some of the other purposes to which they are admirably adapted, and that a block of three small dwelling houses should be converted into a Physical Laboratory. One of these could be leased to the Director, and the other two would contain the large number of small rooms especially required for this work. Among the numerous advantages of this plan are the economy, considering the additional space gained, the means of erecting stone piers to secure steadiness for delicate instruments, the light for spectroscopic and photographic work, the easy access to the ground for out-door experiments, the comparatively unobstructed view for astronomical or meteorological observations, and the convenience for undertaking experiments requiring power.

The work of the Laboratory may be divided into three classes: that for the regular students, additional courses, and original investigation. The first of these has been fully described elsewhere: in previous reports, in the Catalogues of the Institute, and in the "Elements of Physical Manipulation." The latter work, in particular, gives in detail the work of the Laboratory, and is, in fact, based on the manuscript directions to students in performing their various experiments. It will therefore be sufficient here to state, in a general way, what would be the work of an average student in the third year in the course as now given. The actual experiments differ a little in each case, but the following list shows approximately what is done by every regular student in the third year's Class.

3. *Insertion of Cross-Hairs*, teaching the student to handle delicate objects.

11. *Calibration of Water*, for graduating vessels.

12. *Cathetometer*, measuring the height of columns of liquid.

14. *Spherometer*, measuring the curvature of a lens.

15. *Estimating tenths of a Second*, as in very accurate measurements of time.

23. *Composition of Forces*, illustrating the parallelogram of forces.

25. *Parallel Forces*, measuring their resultant and comparing with theory.

28. *Crank motion*, comparing the relative positions of a piston rod and fly-wheel with that given by theory.

35. *Deflection of Beams*, proving the laws of elasticity.

41. *Borda's Pendulum*, determining the force of gravity by the time of vibration of a pendulum, and measuring its length by (20) *Reading Microscopes*.

45. *Hydrometers*, showing how specific gravity is found.

46. *Specific Gravity Bottle*, for measuring specific gravities.

67. *Absorption Photometer*, measuring the light cut off by glass.

69. *Bunsen Photometer*, for testing the quality of illuminating gas.

76. *Spectroscope*, observing and measuring various spectra and analyzing by the spectroscope various unknown mixtures.

78. *Law of Lenses*, comparing their observed and computed foci.

79. *Microscope*, using various appliances, as reflected and oblique illumination, camera lucida, micrometers, etc.

88. *Opthalmoscope*, viewing the interior of an artificial eye, illustrating various diseases.

92. *Polarized Light*, illustrating a subject otherwise acquired by the student only with difficulty.

96. *Telegraph*, teaching the student to send messages.

98. *Law of Galvanometer*, or currents corresponding to various deflections.

102. *Wheatstone's Bridge*, measuring resistances with apparatus like that used on the Atlantic Cable.

103. *Resistance Coils*, making and testing resistances by the British Association Bridge.

105. *Electromotive Force and Resistance of a Battery*, or a complete test of a galvanic battery.

118. *Force of Magnets*, or their attraction measured at various distances.

125. *Expansion of Liquids*, with various temperatures.

132. *Law of Cooling*, or the connection between time and temperature.

133. *Pressure of Steam*, at various temperatures.

138. *Specific Heat*, measuring first that of water.

145. *Efficiency of Gas Burners*, measuring the number of units of heat generated with a given burner per cubic foot of gas burned.

Since this only represents about fifty hours' work, or two hours a week for twenty-five weeks, the apprehension that all the time would be spent on a few experiments, proved groundless. If the means were sufficient to keep the apparatus in perfect order, there is no reason that this amount of work should not be made the minimum required of all students. Several of the more advanced institutions of learning in this country have already successfully adopted our system in the form in which we use it, and at many others its adoption is strongly desired. The only other plan competing with it, is that in which every experiment shall be very exhaustively treated, which requires a long time to be devoted to each one. This system may have advantages for specialists (though even then I think it should follow that given above), but I doubt its advisability with large classes of students not particularly interested in physics as a profession. As a matter of general culture, the list of experiments detailed above would have greater value than any single experiment, however fully treated. The use of the course here described as a means of culture should not be overlooked, as it is with this object that its introduction into our larger colleges is to be expected. A student accustomed to learn merely from books acquires a new knowledge of physical phenomena, when he himself proves the correctness of theoretical laws by actual experiment. Facts thus learned are also far more readily remembered. An interesting feature of this method of teaching is the rapid improvement, especially with classes who have had no previous laboratory practice. Such a class, during their first hour, accomplish almost nothing, and almost discourage both themselves and their instructor; the next hour shows an improvement, and before many weeks, experiments are readily performed without question, which at first were quite unintelligible to them.

A good test of the character of the instruction, is the relation of teacher to pupil. My own relations, which have always been of the pleasantest character, I ascribe largely to the interest of the classes in their work. They remain beyond the hour, encroach

upon their dinner-hour, and frequently ask, and are allowed, to work at other times which they might devote to amusement. With this condition of things disorder is almost unknown, and they are treated, and I believe regard themselves, as friends and guests, rather than as pupils. Of course the consequence shows itself in a largely increased amount of work, and a saving of much of the nervous wear and tear of a teacher's life.

These remarks apply with still more force to the work of the fourth year, which is largely of a professional nature. The students in Civil Engineering and Architecture devote two hours a week to work in the Physical Laboratory for half of their fourth year. The work during the present year serves as a type of that previously done. This year two students planned and built a truss of a form suitable for a roof, and measured its change of shape under various loads. It finally broke under three hundred pounds, while the bars of which it was formed would not have borne a tenth part of the weight. Others studied the laws of continuous girders, and compared the deflections with those given by theory; others tested a water-motor, measuring the flow, the pressure, the work done, the speed and other elements, and from these computed the efficiency. Others, again, measured the strength of wires, the force required to strip a nut off a bolt, and compared the effect of impact, as in a pile-driver, with a dead weight. In former years, several excellent models of bridges have been built by students, and these are now used for tests of change of form with varying loads and for other purposes.

Several other courses have also been given, mostly to students intending to make Physics their profession. Of these the following may be mentioned: —

Mechanical Engineering. Formerly the mechanical engineers spent four hours a week in their fourth year in the Laboratory. With the establishment of a Mechanical Laboratory this work has been given up, and the course is now given to the students in Physics only. It includes various mechanical measurements, as flow of water, strength of materials, velocity of shafting, friction and power (Phys. Manip., II, pp. 109–138).

Lantern Projections. To students in Physics only. The various methods of using the lantern as a means of instruction

(Phys. Manip., II, pp. 212–253). Last year, when several students in the fourth year were devoting themselves to Physics, this course was supplemented by some exercises which proved most successful and useful. Some lectures were given by these students to pupils of some of the neighboring private schools, and illustrated by the lantern and the other lecture apparatus of the department. Thus our students acquired the very best experience to fit them as teachers. The course in Lantern Projections has sometimes been given by Prof. Cross, in connection with his own lectures.

Geodesy. Last year, finding that the students in Civil Engineering received little instruction in this branch of higher surveying, a course of lectures was given to the older students on geodesy and topography, with especial reference to the Geological Surveys now under consideration in many of the United States.

Practical Astronomy. This course, which has been given for the last two years in the form of lectures, promises, by its adoption, an excellent opportunity to some college, provided with the necessary instruments, for the establishment of an Astronomical Laboratory (Physical Manip., II, pp. 166–212).

Photography. This course, originally given by Mr. Whipple, is now given by Prof. Cross to students in Physics only.

Advanced Physics. One of the most successsful courses for the special students in Physics is that bearing this name. The form of each exercise is that of a scientific meeting, different students being selected in turn as secretaries. The advantages of a student's society are attained, while the objections of irregular attendance, and that the work would fall mainly on two or three members, are obviated by requiring from all, attendance and the presentation of papers. Each student in turn presents papers, often illustrated by experiments, either on some original work, or a review of some recently published research, or of the latest scientific periodicals. Many of the standard researches in physics have been thus presented, and other matters of value to every scientific man, such as the character of various scientific societies and periodicals and other similar matters. This, with the lectures referred to above, qualifies each student to express himself clearly and composedly before an audience.

For the past two years instruction has been given in the Physical Laboratory to the students of the Boston University, with results highly satisfactory to us. The laboratory course has thus been successfully tested with a class whose previous preparation has been literary rather than scientific. The income derived could, moreover, have ill been spared in maintaining the Laboratory.

The great object to which my work for these last ten years has been directed, has been original investigation. This is the real goal to which the attention of a student in the Physical Laboratory is directed. The value of the course in physical manipulations is largely that it teaches a student to think for himself, and prepares him with methods by which he may solve any problem for himself experimentally. I have endeavored to impress on all our students in physics the principle that original investigation should be the great aim of every scientific man. In consequence, a great deal has been done in the laboratory that is new, but in many cases there has been neither time nor means for publication. The following articles have been published under the titles here given. Where I have aided the student largely in the work, my name appears; in the other cases the work has been done almost entirely by the student.

On the Focal Length of Microscope Objectives. By Chas. R. Cross. *Journ. Frank. Inst.*, lix, 401 (June, 1870).

On the Relative Efficiency of Kerosene Burners. By Chas. J. H. Woodbury. *Journ. Frank. Inst.*, xcvi, 115 (Aug., 1873).

The Phonautograph. By Chas. A. Morey. *Amer. Journ. Sci.*, cviii, 130 (Aug., 1874).

I. Foci of Lenses placed obliquely. By Prof. E. C. Pickering and Dr. Chas. H. Williams. *Proc. Amer. Acad.*, x, 300.

II. Light transmitted by one or more Plates of Glass. By W. W. Jacques. *Proc. Amer. Acad.*, x, 389.

III. Intensity of Twilight. By Chas. H. Williams. *Proc. Amer. Acad.*, x, 421.

IV. Light of the Sky. By W. O. Crosby. *Proc. Amer. Acad.*, x, 425.

V. Light absorbed by the Atmosphere of the Sun. By E. C. Pickering and D. P. Strange. *Proc. Amer. Acad.*, x, 428.

VI. Tests of a Magneto-Electric Machine. By E. C. Pickering and D. P. Strange. *Proc. Amer. Acad.*, x, 432; *Electrical News*, i, 14, 54.

VII. Answer to M. Jamin's objection to Ampère's Theory. By W. W. Jacques. *Proc. Amer. Acad.*, x, 445.

VIII. An Experimental Proof of the Law of Inverse Squares for Sound. By Wm. W. Jacques. *Proc. Amer. Acad.*, xi, 265.

IX. Diffraction of Sound. By Wm. W. Jacques. *Proc. Amer. Acad.*. xi, 269.

XI. On the Effect of Temperature on the Viscosity of Air. By S. W. Holman. *Proc. Amer. Acad.*, xii, 41.

The following articles were published in the Report of the Physical Laboratory for the year 1871–72.

The first four are Reports of the Civil Engineers of the Fourth Year in the course referred to above.

Report upon Experiments on a Queen-Post Truss and its component parts. By Messrs. C. S. Ward and B. E. Brewster.

Measurement of the Angular Deflection of Beams fixed at one end. By Mr. C. F. Allen.

Apparatus for ruling lines with a Diamond. By Mr. C. F. Allen.

Coefficients of Efflux. By Messrs. W. B. Dodge and W. E. Sparrow.

Buoyant effect of a column of heated air from a Bunsen burner. By Mr. C. K. Wead.

Law of Lenses. By Mr. C. K. Wead. The focal length of two or more lenses separately and combined, were here measured.

Photometric Experiments. By Mr. C. K. Wead. The light cut off by 1 to 7 plates of glass was measured, also the absorption of ground glass. Another series of experiments gave the light of an Argand burner with various consumptions of gas.

To find the Refractive Formula (that of Cauchy) for any Substance. By Mr. F. W. Very.

Micro-photography. By Mr. C. S. Minot.

Saccharimetry. By Mr. F. A. Emmerton.

Of the unpublished papers may be mentioned the following: —

On Covering Steam Pipes. By several students, collated by Mr. B. H. Locke.

Magnetization by Frictional Electricity. By Mr. C. K. Wead.

Electricity generated by a Holtz Machine in absolute measures. By Mr. S. J. Mixter.

On the Density of the Earth. By Mr. J. B. Henck, Jr.

Measurements of Plateau's Soap-Bubble Films. By Mr. W. E. Nickerson.

Velocity of Air Currents at various distances from an Air-jet. By several students.

Resistance of Water in Pipes. By several students.

Measurement of curvatures and minute distances by Newton's Rings. By Dr. C. H. Williams.

Law of Vibration of a Tuning Fork. By Mr. S. H. Wilder.

Effect of the Intensity of Sound upon its Velocity. By Mr. W. W. Jacques.

It remains for me to speak of my own work, which I do without hesitation; first, since the reputation of the Institute is largely that of those connected with it; and, secondly, because the intellectual as well as the physical results of my work should be in the possession of the Corporation of the Institute.

As the entire work of building up the Physical Laboratory, making or having made the apparatus, and conducting the exercises has been in my hands, unaided by any assistant, I have had less time than I desired for original investigation. Besides this, I have given all the other courses described above, except where the contrary is expressly stated, and, in addition, have delivered four Lowell Courses of lectures. The first of these, on Sound, was in 1869–70; the second, on Experimental Physics, in 1871–2, gave a class composed mainly of teachers, an opportunity to learn the methods of our Laboratory; the third, on the Applications of Electricity and Magnetism, was given at the Lowell Institute in 1873–4; and the fourth, on Lantern Projections, in 1874–5, was especially designed to aid teachers in using this instrument. In 1872 the laboratory was kept open during nearly the entire summer, and instruction in its methods given to a class of half a dozen gentlemen, mostly Professors of Physics in Western colleges. Although the number of regular students in physics has been small, as was to be expected, there have been every year several special students, some of whom devoted almost all

their time to physical investigation. To provide work for them, many of the researches I should otherwise have undertaken myself I have put in their hands, and this proved an additional drain on the limited number of subjects of investigation which will occur to a single individual. My principal work has been the publication of the two volumes entitled "Elements of Physical Manipulation," which embody the experiments performed in the current work of the Laboratory. During the summer of 1869, I accompanied the Nautical Almanac Expedition to Iowa to observe the total eclipse of the sun of August 7th. In 1870, through the liberality of the Government of the Institute, I was enabled to accompany the Coast Survey Expedition to Spain, to observe the total eclipse of the sun of December 22d. The summer of 1873 was largely devoted to observations, with a new form of polarimeter, on the light of the sky and that reflected and refracted by plates of glass. During the summer of 1876 several thousand observations were taken of the heights of the White Mountains with a micrometer level. The results, which will probably supply the most complete survey yet made of this region, are now in process of reduction. The following list gives the references to the published accounts of these researches, together with the more important of my other published papers: —

Dispersion of a Ray of Light refracted at any number of Plane Surfaces. *Proc. Amer. Acad.*, vii, 478 (April, 1868).

Essay on the Comparative Efficiency of different forms of the Spectroscope. *Amer. Journ. Sci.*, xlv, 301 (May, 1868).

Description of a Machine for Drawing the Curves of Lissajous. *Journ. Frank. Inst.*, lvii, 55 (Jan., 1869).

Plan of the Physical Laboratory. (April, 1869.)

A New Form of Spectrum Telescope. *Engin. and Min. Journ.* (July, 1869).

Report on the Total Eclipse of August 7th, 1869. *Journ. Frank. Inst.* lviii, 281 (Oct., 1869). Trans. into French in *Les Mondes*, 1869, 573.

Observations of the Corona during the Total Eclipse. *Phil. Mag.* xxxviii, 281 (Oct., 1869).

Note on the Supposed Polarization of the Corona. *Journ. Frank. Inst.*, lviii, 372 (Dec., 1869).

On the Diffraction produced by the Edges of the Moon. *Journ. Frank. Inst.*, lix, 265 (April, 1870).

Polarization of the Corona. *Nature*, iii, 52 (Dec., 1870).

Spectrum of the Aurora. *Nature*, iii, 104 (Dec., 1870).

List of Observations of the Polarization of the Corona. *Journ. Frank. Inst.*, lxi, 58 (Jan., 1871).

The Graphical Method. *Journ. Frank. Inst.*, lxi, 272 (April, 1871).

Photographing the Corona. *Journ. Frank. Inst.*, lxii, 54 (July, 1871).

On Dispersion, and the Possibility of Attaining Perfect Achromatism. *Proc. Amer. Assoc.*, xix, 62 (Aug., 1871).

The Eclipse of 1870. *Old and New*, iii, 634 (May, 1871).

Report of Observations of Total Eclipse of the Sun of Dec. 22d, 1870. *U. S. Coast Survey Report*, 1870, 115, 229.

Report on the Physical Laboratory. 1871.

A Geometrical Solution of some Electrical Problems. *Journ. Frank. Inst.*, lxvi, 13 (July, 1873).

Applications of Fresnel's Formula for the Reflection of Light. *Proc. Amer. Acad.*, ix, 1 (Oct., 1873).

Measurements of the Polarization of Light reflected by the Sky and by one or more plates of glass. *Amer. Journ. Sci.*, cvii, 102 (Feb., 1874); *Phil. Mag.*, xlvii, 127 (Feb., 1874).

Applications of the Graphical Method. *Proc. Amer. Acad.*, ix, 232 (May, 1874).

Graphical Integration. *Proc. Amer. Acad.*, x, 79 (Oct. 1874).

Mountain Surveying. *Proc. Amer. Acad.*, xi, 256 (Jan., 1876).

Height and Velocity of Clouds. *Proc. Amer. Acad.*, xi, 263 (Jan., 1876).

Comparison of Prismatic and Diffraction Spectra. *Proc. Amer. Acad.*, xi, 273 (June, 1875).

Elements of Physical Manipulation. In 2 Vols. Vol. I, 1873, pp. 225. Vol. II, 1876, pp. 316.

Allow me now to compare the present condition of the Physical Department with that of ten years ago. Then we had only a single room, with a second unfurnished, very little apparatus, and the entire instruction consisted of one course of lectures. Now, besides the lecture-room, we have two large laboratories, one subdivided so as to give five small rooms, and, in addition, rooms for the spectroscope and for photography. The lecture course has

been for some years ably administered by Prof. Cross, and a course of lectures on Descriptive Astronomy has been added to it. The apparatus is in good condition, and contains a large collection of slides for the lantern. A large Physical Laboratory has been built up, which now accommodates a hundred students a year. The course has been extended into the fourth year, giving technical experiments to the architects and civil engineers, and to our own students in photography, lantern projections and practical astronomy. Original investigation is encouraged, and numerous articles have been published by our students in the scientific periodicals of the day. The principles on which our work is conducted are few and simple. First, use is of the first importance, appearance, secondary. All our apparatus is based on this fact. Polish and lacquer are dispensed with, and the money saved is expended on the working parts. Secondly, originality on the part of the student is encouraged to the uttermost, and he is taught that, as a scientific man, original research should be his highest aim. Thirdly, there are no secrets in science, and accordingly every aid has been extended to other institutions desiring to adopt our methods, by giving them the results of our experience.

Of the present needs of the Laboratory the principal one is an endowment to cover running expenses; in this way only can it be placed on a permanent foundation. The apparatus is of the simplest kind, and comprises but few instruments of precision in the modern sense of the term. It has never received the large sum usually expended on original equipment. For the older students, especially in the course in Advanced Physics, a working library of the standard works in physics, and a few of the later scientific periodicals, are much needed. This last need has been in part supplied by the liberality of a friend of the Institute, but where the journals are to be divided among so many departments, each gets but little. Another generous friend has twice given a large sum to the Laboratory, once for a spectroscope, the most powerful ever made at the time of its completion, and, again, by a timely gift, enabled a large sum to be expended on original investigation. Several of the most important papers that have issued from the Laboratory resulted from this gift. The Department is also greatly indebted for a most liberal gift for acoustic apparatus.

Two large sums have been given for this purpose, and have furnished the lecture course with an excellent collection of instruments to illustrate the laws of sound.

I cannot close this report without an acknowledgement of the aid I have received from you, Mr. President, in bringing our Laboratory into its present state of efficiency. Your confidence in its success from the very beginning, your encouragement and enthusiasm regarding its extension, and the interest you have shown in every detail, have helped, more than we have realized, to such success as we have attained.

My thanks are also due to Prof. Cross, whose work as student, assistant and professor, has been wholly devoted to the interests of our department. To his unwearied efforts the present high efficiency of the lecture course is very largely due.

With hopes that the next decade may witness as great advances as that which is just completed, I remain,

Very respectfully yours,

EDWARD C. PICKERING,

Thayer Professor of Physics.

The style is clear and concise, thus concentrating
amount of material in a small space. As it is es
that the beginner should bear in mind all the dire
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aimed at in the earlier parts of the work. The posi
the author in charge of one of the first and largest
ical laboratories in the country, has given him an op
nity he could not elsewhere have obtained to test m
the experiments with large numbers of students.
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employed, thereby eliminating many of the errors,
in a work of its character would otherwise be so di
to avoid.

The preliminary chapter is devoted to general me
of investigation, and the more common applications
mathematics to the discussion of results. The gra
method does not seem to have attracted the atten
deserves; it is accordingly compared here with th
lytical method. Some new developments of it are
over inserted. A short description is also given
various methods of measuring distances, time, and w
which, in fact, form the basis of all physical investi
This chapter is intended as the ground-work of a
course of lectures, given to the students before they
their work in the laboratory. The remainder of the v
is occupied with a series of experiments, and the an
list indicates the range of topics.

GENERAL EXPERIMENTS.

1. Estimation of Tenths.
2. Verniers.
3. Insertion of Cross-hairs.
4. Suspension by Silk Fibres.
5. Temperature Curve.
6. Testing Thermometers.
7. Eccentricity of Graduated Circles.
8. Contour Lines.
9. Cleaning Mercury.
10. Calibration by Mercur
11. Calibration by Water.
12. Cathetometer.
13. Hook Gauge.
14. Spherometer.
15. Estimation of Tenth Second.
16. Rating Chronometers.

MECHANICS OF SOLIDS.

ECHANICS OF LIQUIDS AND GASES.

SOUND.

LIGHT.

75. Index of Refraction.
76. Chemical Spectroscope.
77. Solar Spectroscope.
78. Law of Lenses.
79. Microscope.
80. Preparation of Objects.
81. Mounting Objects.
82. Foci and Aperture of Objectives.
83. Testing Plane Surfaces.
84. Testing Telescopes.
85. Photography I. Glass Negatives.
86. Photography II. Paper itives.
87. Testing the Eye.
88. Opthalmoscope.
89. Interference of Light.
90. Diffraction.
91. Wave Lengths.
92. Polarized Light.
93. Polariscope.
94. Saccharimeter.

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THE RIVERSIDE PRESS, Cambridge.

aid in education. Certain subjects, such as Microscopy, Spectroscopy, Photography, Polarized Light and Electrical Measurements, are treated more in detail for those interested in these specialties.

Librarians and others interested in works of reference are requested to examine this book, as containing many important matters not elsewhere presented in so compact a form.

The chapter on Astronomy shows how this science, like Chemistry and Physics, may be taught by the laboratory method, and suggests a valuable course by which each member of a class may learn to determine the latitude, longitude, and time by a sextant or transit. The tables and descriptions of astronomical objects will also interest those having small telescopes.

The great practical importance of experiments in Mechanical Engineering, and the unsatisfactory nature of the methods often employed, led to the introduction of a chapter on this subject also. The mechanical engineer will here find a description of various methods of making the measurements required for testing the strength of materials or the power of machines.

A preliminary chapter is devoted to the mathematical methods of discussing the results of experiments, and to a description of the various instruments for measuring time, weights, and distances, which in fact form the basis of all physical measurements. Then follows a series of experiments in mechanics of solids, liquids, and gases, sound, light, electricity, magnetism, and heat, forming the main portion of the work. Each experiment is divided into two parts: the first, *Apparatus*, intended mainly for the instructor, gives a description of the instruments to be employed; the second, entitled *Experiment*, directs the student precisely how he is to use the apparatus. In the second volume four kindred subjects — Mechanical Engineering, Meteorology, Astronomy, and Lantern Projec-

tions — are treated in a similar manner. These are followed by three appendices, A, B, and C. Appendix A briefly describes the principles of Electrical Measurements. Appendix B contains a large collection of Mathematical and Physical Tables brought into a very compact space. Appendix C describes in detail the method of establishing and conducting a Physical Laboratory, and shows how this may be done at very small expense.

The great object of the book is to foster experimental research, and it therefore contains much of the practical knowledge needed for such work. As this is beyond the scope of the ordinary text-books, every physicist has heretofore been obliged to acquire it by long and often laborious personal experience.

The style is clear and concise, thus concentrating a large amount of material in a small space. As it is essential that the beginner should bear in mind all the directions for an experiment before performing it, brevity is especially aimed at in the earlier parts of the work. The position of the author, in charge of one of the first and largest physical laboratories in the country, has given him an opportunity he could not elsewhere have obtained to test most of the experiments with large numbers of students. The work is, in fact, based on the manuscript directions thus employed, thereby eliminating many of the errors, which in a work of its character would otherwise be so difficult to avoid. The annexed list indicates the range of topics.

VOLUME I.

PHYSICAL INVESTIGATION. Analytical Method; Graphical Method; Measurement of Time, Weights, and Distances.

GENERAL EXPERIMENTS. Twenty-two Experiments, including Estimation of Tenths, Insertion of Cross-hairs, Calibrations, Making Weights, and Spherometer.

MECHANICS OF SOLIDS. Twenty Experiments, including Composition of Forces, Friction, Elasticity, and Pendulums.

a.l.